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Fourier Transformation on Model Fitting for Pakistan Inflation Rate

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Abstract

Inflation is one of the serious economic indicators in Pakistan. Inflation can be crawling, walking, running, hyper and stagflation according to nature. To model monthly inflation rate in Pakistan periodogram analysis and frequency domain analysis which is also known as Fourier analysis or spectral analysis is used. After analyzing the data, inflation cycle length is observed and appropriate Fourier series models are fitted to the data. Monthly inflation rate is also analyzed by Auto Regressive Integrated Moving average (ARIMA). Further, models are compared and it is found that Fourier series models are more suitable to forecast inflation rate of Pakistan.

Keywords: Inflation, Periodogram, Fourier Series Models, ARIMA Models, Stationarity, Root Mean Square Error, Mean Absolute Error

1. Introduction

Inflation is a situation in which price level increases and purchasing power decreases. There are certain elements cause the rise of inflation such as increase in population, increase in demand, lack of supply, development expenditures and constant low production due to some social, political, climatic, national or international situations. Increase in inflation rate gives birth to unemployment, poverty, unfair distribution, poor labour force and social evils. So, for economic growth of a country, it is important to analyze inflation rate of a country.

Time series is the collection of observation with respect to time or space. Time series analysis is also known as time domain analysis. Box and Jenkins (1976) methodology come under time domain analysis. Time series may contain trend, seasonality, cyclic and random component. Therefore it is important to explore the stationarity of data. A stationary series has a wave-like pattern as it has constant mean and constant variance. Waves have a period which is the distance between peaks or time between two peaks of a wave. Frequency is the closely related property of the wave period and is just the proportion of cycle that occurs during one observation. Frequency domain is used to analyze the data with respect to frequency.

Transformation is very important in frequency domain analysis and is used to convert a time domain function into a frequency domain. Fourier transformation found by great mathematician Joseph Fourier (1822) is the most commonly used transformation in frequency domain analysis. This frequency domain analysis is also known as spectral analysis or Fourier analysis. In this paper frequency domain approach with periodogram analysis is used to model the monthly inflation rate. Model based on spectral analysis and model based on Box-Jenkins methodology are also compared.

2. Literature Review

Etuk (2012) presented time series analysis of Nigerian monthly inflation rates and fitted a multiplicative seasonal autoregressive integrated moving average model. He showed that the model is adequate and forecasts are agreed closely with observations. Raza, Javed and Naqvi (2013) discussed the impact of inflation on economic growth of Pakistan and estimated short run and long run relationship between inflation and economic growth. They suggested that government should maintain inflation in single digit which is favorable for economic growth.

Ekpenyong, John, Omekara and Peter (2014) proposed the application of periodogram analysis and Fourier analysis to model inflation rates in Nigeria. The main objective of the study was to identify inflation cycles and fit the appropriate model to forecast future values. Konarasinghe and Abeynayake (2015) suggested a study based on Fourier transformation on model fitting for Sri Lankan share market. They also analyzed the monthly returns by

ARIMA (Auto Regressive Integrated Moving Average) and concluded that Fourier transformation along with multiple regression is suitable.

Konarasinghe, Abeynayake and Gunaratne (2015) proposed a model by using Box-Jenkins methodology or Auto-Regressive Integrated Moving Average (ARIMA) methodology to forecast Sri Lankan share market returns. Jere and Siyanga (2016) used Holt's exponential smoothing and Auto-Regressive Integrated Moving Average (ARIMA) models to forecast inflation rate of Zambia. They showed that the choice of Holt's exponential smoothing is as good as an ARIMA model.

Konarasinghe, Abeynayake and Gunaratne (2016) developed a model named as circular model based on Fourier transformation to forecast Returns of Sri Lankan share market. Thomson and Vuuren (2016) proposed Fourier transform analysis to determine the duration of South African business cycle which is measured by using log changes in nominal GDP (Gross Domestic Product). Three dominant cycles are used to forecast log monthly nominal GDP and the forecasts are compared with historical data. They found that Fourier analysis is more effective in estimating the business cycle length as well as in determining the position of the economy in the business cycle.

3. Research Methodology

The main objective of this study is to develop a significant model to forecast inflation rate in Pakistan. For this purpose, monthly data on CPI inflation rate from 2008 to 2016 is collected from (Pakistan Bureau of Statistics, n.d.). First periodogram analysis is used to find inflation cycle as inflation is affected by many factors which may cause seasonality and periodicity in the series. After determining the period or frequency of series, significance of selected period is tested by a test developed by Fisher (1929). Fourier series is then used to model inflation rate by utilizing selected frequency as Fourier frequency. Moreover, Box-Jenkins methodology is also used to model the data. By using accuracy measures both types of models are compared and it is found that model based on Fourier series is considering smaller deviation in Root Mean Square Error (RMSE), Mean Absolute Error (MAE) and Mean Absolute Percentage Error (MAPE).

3.1 Periodogram Analysis

Periodogram is a tool that partitions the total variance of a time series into component variances like ANOVA. The longer cycle shares large variance in the series. In general practice periods of cycles are not known then Periodogram is utilized to identify dominant cyclic behavior in the series. In periodogram analysis, time series can be viewed as

$$Y_t = T_t + \sum_{i=1}^{N} \left(a_i \cos w_i t + b_i \sin w_i t \right) + \varepsilon_t , \qquad (1)$$

where T_t is trend, N is total number of observations, a_i and b_i are coefficients, w_i is angular frequency in radians and ε_t is error term. The coefficients are calculated as

$$a_i = \frac{2}{N}\sum_{t=1}^{N}\left(Y_t - \hat{T}_t\right)\cos w_i t, \tag{2}$$

$$b_i = \frac{2}{N}\sum_{t=1}^{N}\left(Y_t - \hat{T}_t\right)\sin w_i t. \tag{3}$$

The calculated coefficients are then used to obtain Intensity or Periodogram ordinate at frequency f_i as

$$I(f_i) = \frac{2}{N}\left(a_i^2 + b_i^2\right) \qquad i = 1,\ 2,.\ q, \tag{4}$$

In case of even number of observations $N = 2q$, $q = N/2$ and for odd number of observations $N = 2q+1$, $q = (N-1)/2$. The period against the largest Intensity that is actually the largest sum of squares is selected as cycle period of series.

3.2 A Significance Test for Periodic Component

The variability in the sizes of sum of squares may be due to just sampling error. The largest ordinate must indicate strong periodicity even for white noise series. Therefore it is necessary to test the significance of largest periodic component in white noise. A test developed by Fisher (1929) provides a reasonable method for testing significance of such periodic components. To perform Fisher test g statistic is computed which is the ratio of largest sum of squares (or intensity ordinate) to the total sum of squares. Tables of critical values for this test statistic are given in Russell (1985). The null hypothesis of white noise series is rejected, if the value of g statistic is greater than critical value.

3.3 Spectral Analysis

The basic idea of spectral analysis is to transform the time domain series into frequency domain series, which determines the importance of each frequency in the original series. This target is achieved by using Fourier transformation.

The general Fourier series model that contain components of time series is given by

$$Y_t = T_t + \sum_{i=1}^{k}\left(\alpha_i \cos iwt + \beta_i \sin iwt\right) + \varepsilon_t, \tag{5}$$

where

$w = 2\pi f$

$k = n/2$ (In case of even number)

$k = (n-1)/2$ (In case of odd number)

n is the number of observations per season or cycle length, f is the Fourier frequency or number of peaks in series, k is the harmonic of w (see Delurgio, 1998), t is the time index, α_i

and β_i are amplitudes which are estimated through multiple linear regression analysis.

3.4 ARIMA

Autoregressive integrated moving average models are presented by Box, Jenkins and Reinsel (1994). To achieve stationarity, if a series is differenced d times then the model will be ARIMA (p,d,q), where p denotes autoregressive terms and q denotes moving average terms. ARIMA model for a stationary time series is a linear equation in which predictors are lags of dependent variable and lags of error terms i.e.

$$\phi_p(B)\Delta^d Y = \theta_q(B)\varepsilon_t, \qquad (6)$$

where, Y_t = present value, d = difference, B = backshift operator, ε_t = present error term

4. Results and Discussion

Time series plot of original inflation rate in Pakistan is constructed. Figure 1 shows that there exist trend, seasonality and cyclical variations in the series but the length of the cycle is not confirmed.

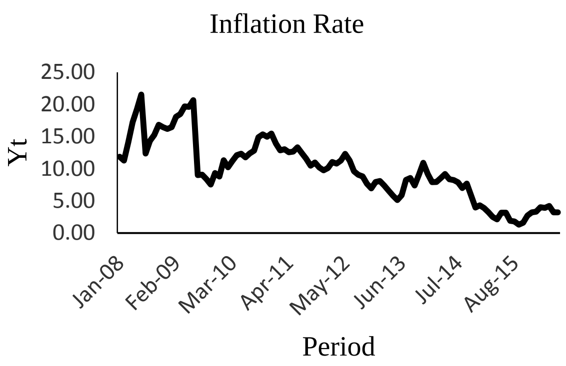

Figure 1. Original Series of Inflation Rates

In this study first stationarity of the series is examined by using ACF (Autocorrelation Function), PACF (Partial Autocorrelation Function) and ADF (Augmented Dicky Fuller) test and it is found that series is not stationary. As figure 1 indicates there exist trend therefor to make such a time series stationary, the trend is estimated and then removed from the series.

4.1 Fitting Trend Model

The estimated trend equation is given by

$$\hat{T}_t = 16.927 - 0.1387t$$

Table 1. Summary of trend model

Predictor	coef	SE coef	t	P
Constant	16.9270	0.4992	33.91	0.000
T	-0.1387	0.0084	-16.48	0.000
S = 2.50222	R-Sq = 73.1%	R-Sq(adj) = 72.8%		

Table 1 shows that both the parameters in the trend model are significant.

4.2 Periodogram Analysis

After removing trend from the series, de-trended series is used to estimate seasonal component. By using equation (2), (3) and (4) intensities at various frequencies are obtained as given in Table 2.

Table 2. Frequencies, Periods and Intensities

freq	P	Pdg	freq	P	Pdg
0		0	0.254902	3.923077	2.469466
0.009804	102	17.82893	0.264706	3.777778	2.571481
0.019608	51	0.283872	0.27451	3.642857	4.361273
0.029412	34	151.7146	0.284314	3.517241	3.985825
0.039216	25.5	5.061672	0.294118	3.4	1.131958
0.04902	20.4	143.5882	0.303922	3.290323	0.115284
0.058824	17	24.33793	0.313725	3.1875	0.848861
0.068627	14.57143	6.144949	0.323529	3.090909	4.099034
0.078431	12.75	13.55495	0.333333	3	4.243186
0.088235	11.33333	17.27606	0.343137	2.914286	3.418537
0.098039	10.2	27.54437	0.352941	2.833333	1.415018
0.107843	9.272727	63.83039	0.362745	2.756757	1.373335
0.117647	8.5	3.69523	0.372549	2.684211	1.026038
0.127451	7.846154	1.240284	0.382353	2.615385	1.869164
0.137255	7.285714	6.553623	0.392157	2.55	3.213267
0.147059	6.8	4.288788	0.401961	2.487805	3.215139
0.156863	6.375	17.12196	0.411765	2.428571	2.99018
0.166667	6	23.81014	0.421569	2.372093	2.466381
0.176471	5.666667	14.43552	0.431373	2.318182	2.288237
0.186275	5.368421	4.424012	0.441176	2.266667	0.927199
0.196078	5.1	3.554464	0.45098	2.217391	0.390481
0.205882	4.857143	3.682635	0.460784	2.170213	0.329865
0.215686	4.636364	0.798868	0.470588	2.125	1.620553
0.22549	4.434783	6.765822	0.480392	2.081633	3.01465
0.235294	4.25	2.518789	0.490196	2.04	3.122662
0.245098	4.08	3.450255	0.5	2	4.191004

Table 2 shows that period and frequency corresponding to largest intensity are n = 34 and f = 0.02941. Here frequency is just the inverse of period. Thus the period of cycle or long-term inflation cycle is 34 months. This shows that inflation rate is high during 2008-01 to 2010-10. The short-term inflation cycle identified by second largest intensity is 20 months that is from 2010-11 to 2013-06.

4.3 Significance Test of Periodic Component

As Fisher test statistic value in Table 3 is greater than the critical value so the null hypothesis of white noise is rejected and the selected period is significant.

Table 3. Output of Significance test

	G statistic	Critical value
Fisher Test	0.2415	0.126

4.4 Estimation of Seasonal Component

Since the number of observations is even therefore,

$$K = \frac{n}{2} = \frac{34}{2} = 17$$

and

$$w = 2 \times \pi \times 0.02941$$

Hence the seasonal model is given by

$$Y_t = \sum_{i=1}^{17} (\alpha_i \cos iwt + \beta_i \sin iwt)$$

The parameters of this model are estimated by the least square method.

Table 4. Parameter Estimation in seasonal component

Predictor	Coef	SE coef	t	p	Predictor	Coef	SE coef	t	p
coswt	0.2011	0.3326	0.60	0.547	sin9wt	-0.1804	0.3326	-0.54	0.589
sinwt	1.7138	0.3325	5.15	0.000	cos10wt	0.0979	0.3326	0.29	0.769
cos2wt	0.5942	0.3326	1.79	0.078	sin10wt	-0.1098	0.3327	-0.33	0.742
sin2wt	-0.3602	0.3325	-1.08	0.283	cos11wt	0.1520	0.3326	0.46	0.649
cos3wt	-0.5690	0.3326	-1.71	0.092	sin11wt	-0.2342	0.3328	-0.70	0.484
sin3wt	0.0970	0.3325	0.29	0.771	cos12wt	0.1696	0.3326	0.51	0.612
cos4wt	0.2673	0.3326	0.80	0.424	sin12wt	0.0322	0.3330	0.10	0.923
sin4wt	-0.0619	0.3325	-0.19	0.853	cos13wt	0.1959	0.3326	0.59	0.558
cos5wt	-0.1403	0.3326	-0.42	0.674	sin13wt	-0.0045	0.3332	-0.01	0.989
sin5wt	-0.2481	0.3325	-0.75	0.458	cos14wt	-0.1704	0.3326	-0.51	0.610
cos6wt	0.4640	0.3326	1.39	0.168	sin14wt	0.1847	0.3339	0.55	0.582
sin6wt	-0.2661	0.3325	-0.80	0.426	cos15wt	0.1041	0.3326	0.31	0.755
cos7wt	0.0541	0.3326	0.16	0.871	sin15wt	-0.0645	0.3356	-0.19	0.848
sin7wt	-0.2593	0.3326	-0.78	0.438	cos16wt	-0.0459	0.3325	-0.14	0.891
cos8wt	0.2135	0.3326	0.64	0.523	sin16wt	-0.1154	0.3450	-0.33	0.739
sin8wt	0.0808	0.3326	0.24	0.809	cos17wt	0.4074	0.4954	0.82	0.414
cos9wt	-0.1208	0.3326	-0.36	0.717	sin17wt	28.55	47.13	0.61	0.547

Table 4 indicates that sinwt is significant at 5% level of significance and sinwt, cos2wt, cos3wt are significant at 10% level of significance. So the estimated seasonal models are given by

$$\Delta\hat{Y}_t = 1.7138 \sin wt \qquad (7)$$

$$\Delta\hat{Y}_t = 1.7138 \sin wt + 0.5942 \cos 2wt - 0.5690 \cos 3wt \qquad (8)$$

Since $\Delta\hat{Y}_t = Y_t - T$

By exploring the randomness of error term with the help of ACF and PACF it is found that error term is not random. So first order autoregressive is used.

Table 5. Summary of first order autoregressive

Predictor	Coef	SE coef	t	P
ε_{t-1}	0.65893	0.07242	9.1	0
s = 1.41653				

Table 5 presents that the parameter estimate of error term is significant in model.

$$\hat{\varepsilon}_t = 0.658822\varepsilon_{t-1} \qquad (9)$$

4.5 General Fourier Series Model

The general models which consist estimated trend, estimated seasonal component and error component may be

$$\hat{Y}_t = 16.927 - 0.1387t + 1.7138 \sin wt + 0.65893\varepsilon_{t-1} \quad (10)$$

$$\hat{Y}_t = 16.927 - 0.1387t + 1.17138 \sin wt + 0.5942 \cos 2wt - 0.5690 \cos 3wt + 0.65893\varepsilon_{t-1} \quad (11)$$

Model (10) and model (11) are found to be overall significant. The residuals of both models are also found to be independently and normally distributed.

Table 6. Summary of Fourier series Models

Model	RMSE	MAE	MAPE
Model (10)	1.68	1.16	15%
model (11)	1.57	1.03	13%

Table 6 shows that RMSE, MAE and MPE for both models are small.

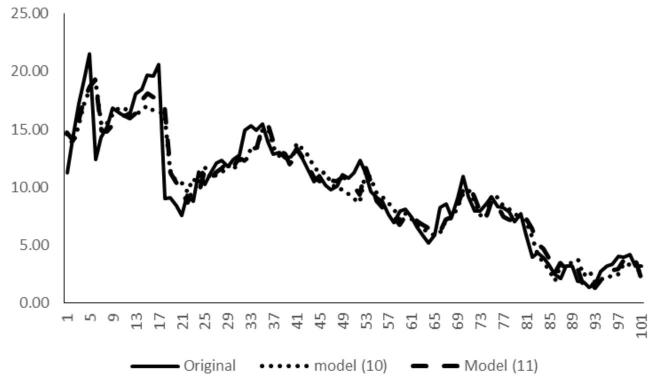

Figure 2. Plot of actual and estimates

Figure 2 reveals that estimated values of both models are very close to actual values.

4.5 ARIMA Models on Inflation Rate

Time series plot of inflation rate shows that series is non-stationary. After taking the first difference series becomes stationary and stationarity is confirmed through correlogram and ADF test. The identification of AR and MA is done through correlogram and ARIMA models of different orders are fitted to the data as given in Table 7.

Table 7. Summary of ARIMA Models

Model	RMSE	MSE	MAPE
(12,1,16)	2.52	1.94	34%
(16,1,16)	3.03	2.20	40%

ARIMA models are found to be overall significant. RMSE, MSE and MAPE of both models are small. The residuals of both models are also independently and normally distributed. From the output of Table 6 and Table 7, it can be seen that Fourier series models are good fitted as compare to ARIMA models.

4. Conclusion

Using periodogram analysis to study inflation rate in Pakistan, it has been found that inflation is much affected by periodic or cyclical variation as long-term inflation cycle is 34 months and short-term inflation cycle is 20 months for the period under study. Fourier series models have been developed to forecast inflation rate in Pakistan. ARIMA models have also been fitted to the monthly inflation rate in Pakistan. It has been observed that accuracy measures RMSE, MAE, MAPE for both types of fitted models are good and residuals are normally and independently distributed. So both methods can be used to forecast Pakistan inflation rate but it is identified that Fourier series models have more effective accuracy measures as compare to ARIMA models.

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