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Abstract

In spite of the abilities of earned value management technique (EVM) in evaluating the projects’ cost and time performance, it is not able to estimate the projects’ future performance. In fact, the earned value management estimates the project future performance merely based on its past and does not consider the environmental circumstances change or other elements which effect on the future performance of the project. Unlike the earned value management, the risk management technique looks to the farther horizons and deals with the evaluation of the project performance by recognizing the opportunities and threats. In this paper we intend to present the risk ratio in combination with the earned value management current indices for better forecasting of the projects’ future performance using the risk management technique. These ratios and indices are determined under real-life and uncertain conditions using the fuzzy approach.

Keywords: Fuzzy earned value, Risk management, Opportunities and threats, Fuzzy risk ratio, Uncertainty
1. Introduction

The earned value management is a project management technique used to measure the projects’ schedule and cost performance within a single integrated methodology (PMI, 2005). This technique assists managers in estimating the final cost and time of the projects. Today a decrease in cost and time is important to all the managers. They should possess a tool and a model for valid forecasts. In this technique the most important elements of a project, i.e. time, cost and scope, must be under control of the project manager. Hence, there is a necessity of noticeable improvements for desired development of earned value management.

According to the Project Management Institute (PMI, 2005), when the earned value management properly applied, it can provides managers with early information and guidelines of performance problems. Earned value management results can be easily transferred to senior management. Thus the project management determines which projects need minor corrective programs and which projects need fundamental ones in order to solve the problems (Gowan, Mathieu & hey, 2004).

Despite being widely believed that implementing the earned value management technique has many advantages, the research on the EVM technique is very limited. Anbari (2003) applied planned value technique (PV) to estimate completion time and improved the earned value (EV) efficiency. Jacob (2003) described the earned duration method (ED) to estimate completion time. Recognizing the EV limitations in determining the schedule performance index (SPI), Lipke (2003) introduced the earned schedule (ES) for $\text{SPI}(t)$ determination. The $\text{SPI}(t)$ has preference over the SPI, because near the end of the project, since the EV approaches to the PV, the SPI index converges to 1, and finally since the EV equals to the PV, at the end of the project, the SPI is always equal to 1. Lipke’s studies were followed by Henderson (2003, 2004) and Vandevoorde and Vanhoucke (2005), and they developed the ES and made it more reliable and applicable by more considerations. Kim et al. (2003) applied the EV in different types of projects and organizations in their research.

Lipke et al. (2009) used the statistical method for forecasting the final time and cost of projects. They studied on 12 companies’ data and proceeded to evaluate their model in different confidence levels and completion percentages. Moslemi Naeni and Salehipour (2011) proceeded to introduce fuzzy earned value indices. They believed that the earned value related data comes from people’s judgments in some projects, since these people’s judgments carry some degree of uncertainty, bringing this reality into interpretations helps in improving the applicability of the EV under the uncertain conditions.

The aim of this paper is to improve the forecasting of projects’ results. Doing this, projects’ managers can possess better information for plans related to projects’ problems. The rest of this paper is organized as following: In section (2) we describe the fuzzy earned value management. The risk management and the way of calculating the fuzzy risk index are discussed in section (3). In section (4) ranking methods of fuzzy numbers and section (5) is related to the research methodology. For better clarification, an example is given at the end.
2. Fuzzy Earned Value Management Technique

Earned value is the budgeted cost of work performed and is evaluated through various techniques depending on the activities’ product and their duration that is illustrated in Table 1.

Table 1. The earned value measurement technique

<table>
<thead>
<tr>
<th>Activities’ product</th>
<th>Activity duration</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1 or 2 periods</td>
</tr>
<tr>
<td>Tangible</td>
<td>-Fixed formula</td>
</tr>
<tr>
<td></td>
<td>-Percent complete</td>
</tr>
<tr>
<td>Intangible</td>
<td>-Level of effort</td>
</tr>
</tbody>
</table>

Here, we only explain the percent complete technique which forms the basis of fuzzy earned value. In this technique, one of the simplest methods for measuring the earned value, in each measurement period the person-in-charge estimates the percentage of the activity completed. In other words, this method arises in situations that the total amount of work required for completion is unknown. The percent estimate mostly is subjective and there is a probability of error and wrong subjective judgment occurrence. For instance, in a cement kilns annual repairs project, the total amount of work required to clean the kilns cannot be certainly stated. In such conditions, according to Moslemi Naeni and Salehipour’s research (2011), using the fuzzy earned value indices is appropriate. Hence, linguistic terms are used for determining the completion percent of each activity. In other words, in case we cannot state an activity progress in certainty, we will use linguistic terms such as very low, half, etc. Fuzzy approach is a normal technique invented by Zadeh in 1960 that is used for forecasting, decision making and activities’ control in environments determined by vagueness, uncertainty and subjectivism (Zadeh, 1965). Linguistic terms applied in EVM technique must convert into fuzzy numbers. To this end, questionnaire is used for determining the fuzzy numbers. These questionnaires are distributed to the project experts and masters in order to converting the linguistic terms into fuzzy numbers in accordance with their useful knowledge and experience about the project activities. For this purpose, triangular and trapezoidal fuzzy numbers are utilized. Linguistic terms and fuzzy numbers are illustrated in Fig. 1. and Table 2. These terms and numbers are only examples.
Figure 1. A fuzzy membership including triangular and trapezoidal fuzzy numbers and the corresponding linguistic terms

Table 2. The assigned fuzzy numbers to each linguistic term

<table>
<thead>
<tr>
<th>Linguistic terms</th>
<th>Fuzzy numbers</th>
</tr>
</thead>
<tbody>
<tr>
<td>Very low</td>
<td>[0,0,0.1,0.2]</td>
</tr>
<tr>
<td>Less than half</td>
<td>[0.1,0.2,0.2,0.3]</td>
</tr>
<tr>
<td>Low</td>
<td>[0.2,0.3,0.4,0.5]</td>
</tr>
<tr>
<td>Half</td>
<td>[0.4,0.5,0.5,0.6]</td>
</tr>
<tr>
<td>More than half</td>
<td>[0.5,0.6,0.7,0.8]</td>
</tr>
<tr>
<td>High</td>
<td>[0.7,0.8,0.8,0.9]</td>
</tr>
<tr>
<td>Very high</td>
<td>[0.8,0.9,1,1]</td>
</tr>
</tbody>
</table>

For instance, the linguistic term “High” equals to the fuzzy number \([0.7,0.8,0.8,0.9]\). A trapezoidal fuzzy number can be considered as \(A=[a,b,c,d]\). Its membership function\(^1\)

\[
\mu_A(x) = \begin{cases} 
0, & x < a \\
\frac{(x-a)}{(b-a)}, & a < x < b \\
1, & b < x < c \\
\frac{(x-c)}{(d-c)}, & c < x < d \\
0, & x > d 
\end{cases}
\]

If \(a=b=c=d\), “A” cannot be called a fuzzy number anymore.

---

\(^1\) The membership degree \(\mu_A(x)\) quantifies the grade of membership of the element \(x\) to the fuzzy set \(A\).
The fuzzy earned value of activity $i$ is illustrated as below which itself is calculated based on fuzzy completion percent and planned budget at completion of the activity $i$:

$$\tilde{EV}_i = \tilde{\epsilon}_i \ast BAC_i = [EV_{i1}, EV_{i2}, EV_{i3}, EV_{i4}].$$  \hspace{1cm} (1)$$

$$\tilde{\epsilon}_i = [a_{i1}, a_{i2}, a_{i3}, a_{i4}].$$ \hspace{1cm} (2)

The $\tilde{\epsilon}_i$ is the fuzzy completion percent of the activity $i$ which is assigned to each activity.

The $BAC_i$ shows the planned budget to complete the activity $i$. The total amount of fuzzy earned value in each measurement period is calculated as below:

$$\tilde{EV} = \sum_{i=1}^{n} \tilde{EV}_i = [\sum_{i=1}^{n} EV_{i1}, \sum_{i=1}^{n} EV_{i2}, \sum_{i=1}^{n} EV_{i3}, \sum_{i=1}^{n} EV_{i4}] = [EV_1, EV_2, EV_3, EV_4].$$ \hspace{1cm} (3)

### 2.1 Fuzzy Performance Indices

Cost performance index (CPI) indicates the efficiency of the project team in utilizing the project financial resources and can be obtained by dividing the earned value by the actual cost:

$$\tilde{CPI} = \tilde{EV}/AC = \left[ \frac{EV_1}{AC}, \frac{EV_2}{AC}, \frac{EV_3}{AC}, \frac{EV_4}{AC} \right].$$ \hspace{1cm} (4)

Schedule performance index (SPI) indicates the efficiency of the project team in utilizing the time and can be obtained by dividing the earned value by the planned value:

$$\tilde{SPI} = \tilde{EV}/PV = \left[ \frac{EV_1}{PV}, \frac{EV_2}{PV}, \frac{EV_3}{PV}, \frac{EV_4}{PV} \right].$$ \hspace{1cm} (5)

The critical ratio (CR) or the schedule cost index (SCI) introduces the efficiency of the project team in effective use of financial resources and time simultaneously and can be

---

2 In the EV indices, if SPI < 1, SPI > 1, and SPI = 1, then the project is behind the schedule, the project is ahead of the schedule and the project is on the schedule, respectively. The same interpretation applies to the CPI except that the CPI deals with the cost, hence if CPI < 1 (>1 or =1) then the project is over (under or within) the budget, respectively.

3 For example, CR = 1 implies that the overall project performance is on target.
indicated as below:

$$\hat{C_R} = \hat{SCI} = \hat{CPI} * \hat{SPI} = \left[ \frac{EV_i^2}{AC * PV} - \frac{EV_i^2}{AC * PV} - \frac{EV_i^2}{AC * PV} - \frac{EV_i^2}{AC * PV} \right].$$ (6)

Several researches have been carried out on the estimated duration at completion of projects (Anbari, 2003; Jacob, 2003; Lipke, 2003). The study about comparing different methods for estimating duration at completion of projects done by Vandevoorde and Vanhoucke (2005), shows that Lipke’s method, i.e. using the earned schedule, has a great efficiency in all cases.

Earned schedule (ES) first introduced by Lipke is in fact the time equivalent of the earned value. Fig. 2. illustrates the earned value concept versus the earned schedule. Earned schedule is calculated as below:

![Figure 2. The earned schedule versus the earned value](image)

$$ES = N + \left( \frac{EV - PV_N}{PV_{N+1} - PV_N} \right).$$ (7)

In the Eq. (7), $N$ is the longest evaluation period in which the current $EV$ is more than $PV_N$. $PV_N$ is the planned value at period $N$, and $PV_{N+1}$ is the planned value of the period after period $N$ (Vandevoorde & Vanhoucke, 2005). The $\pm$, and $ES$ can be calculated as below:

$$ES_i = N_i + \left( \frac{EV_i - PV_{N_i}}{PV_{N_{i+1}} - PV_{N_i}} \right).$$ (8)

$$\pm \{ES_1, ES_2, ES_3, ES_4\}.$$ (9)
In contrast to the SPI, \( \text{SPI}_{(t)} \) is expressed that can be obtained by amount of earned schedule to the actual duration (AD) or \( \text{SPI}_{(t)} = \frac{ES}{AD} \). Its fuzzification is:

\[
\tilde{\text{SPI}}_{(t)} = \frac{\tilde{ES}}{\tilde{AD}} = \left[ \frac{ES_1}{AD}, \frac{ES_2}{AD}, \frac{ES_3}{AD}, \frac{ES_4}{AD} \right]. \quad (10)
\]

### 2.2 Fuzzy Estimate at Completion

The earned value management uses every index calculated in the previous section to estimate the cost and time at completion of project. In fact, the earned value management assumption for estimating cost and time at completion of project is that the future performance of the project is a function of its past. In other words, current calculated operational indices will also remain intact in the future. Nevertheless, we have categorized the existing formulas for fuzzy cost and schedule at completion with different assumptions below.

**I. Fuzzy cost estimate at completion (EAC)**

- **Assuming the CPI is constant to the end of the project**, the fuzzy cost estimate at completion (EAC) will be obtained as below:

\[
\bar{E}_{AC} = AC + \frac{BAC + \hat{EV}}{CPI} = [AC + \frac{BAC - EV}{CPI_1}, AC + \frac{BAC - EV}{CPI_2}, AC + \frac{BAC - EV}{CPI_3}]. \quad (11)
\]

- **If the project completion cost is influenced by the current cost and schedule performance indices**, we will use the following formula:

\[
\bar{E}_{AC} = AC + \frac{BAC - \tilde{EV}}{SCI} = [AC + \frac{BAC - EV}{SCI_1}, AC + \frac{BAC - EV}{SCI_2}, AC + \frac{BAC - EV}{SCI_3}]. \quad (12)
\]

**II. Fuzzy schedule estimate at completion (\( EAC_{(t)} \))**

The performance factor (PF) used here depends on the project status:

- **PF=1**: If it is supposed that the duration of the remained activities makes progress according to the plan, the fuzzy time estimate at completion (\( EAC_{(t)} \)) will be obtained as below:

\[
\bar{E}_{AC_{(t)}} = AD + \left( \frac{PD - \tilde{ES}_1}{1} \right) = \left[ AD + \left( \frac{PD - ES_1}{1} \right), AD + \left( \frac{PD - ES_2}{1} \right), AD + \left( \frac{PD - ES_3}{1} \right) \right]. \quad (13)
\]

- **PF=SPI_{(t)}**: Assuming the duration for the remained activities makes progress based on the \( \text{SPI}_{(t)} \) trend, the fuzzy \( EAC_{(t)} \) will be obtained as below:
\[
\bar{EAC}_{(t)} = AD + \frac{PD - ES}{SPI_{(t)}} \bar{AD} + \frac{PD - ES}{SPI_{(pp)}} \bar{AD} + \frac{PD - ES}{SPI_{(pp)}} \bar{AD} + \frac{PD - ES}{SPI_{(pp)}} \bar{AD}.
\]

- \(PF = SCI(0)\): Assuming the duration for the remained activities makes progress base on the \(SCI(0)\) trend, the fuzzy \(EAC_{(0)}\) will be obtained as below:

\[
\bar{EAC}_{(0)} = AD + \frac{PD - ES}{SCI_{(0)}} \bar{AD} + \frac{PD - ES}{SCI_{(0)}} \bar{AD} + \frac{PD - ES}{SCI_{(0)}} \bar{AD} + \frac{PD - ES}{SCI_{(0)}} \bar{AD}.
\]

3. Risk Management and Fuzzy Risk Index Calculation

The risk management has a profound sight on the depth of the elements which affect the project efficiency that this information is remarkably valuable in assisting the management work. These risks or uncertainties of future conditions of the project include both opportunities and threats which may cause some ambiguities and changes in the project future performance. Regardless of the number and definition of stages, the project risk management processes have one common element: “an activity that deals with planning actions that will be implemented in order to reduce Being exposed to risk” (Ben-David, Raz, 2001). This principle activity can be subdivided into four major stages: planning, identifying risk, analyzing risk, and response to it. By using risk analysis a project manager evaluates quantitatively or qualitatively the likely consequences of risks as well as the likelihood of occurrence (Raftery, 1994).

Considering the project performance forms under the influence of its constituent elements, we should take steps to provide a list of elements which affect the project performance. Managers can use risk breakdown structures (RBS) for providing this list. The risk breakdown structure will assist in recognizing the plentiful uncertainties of different areas of the project and in taking step for forecasting the future conditions of the project in all of those areas. Thus, a list of opportunities and threats which may occur during the project fulfillment is collected. The more perfect and comprehensive the breakdown structure used in risk breakdown or recognition level of elements and constituent dimensions of the project performance is, the more recognition of changes in the future conditions of the project can be trusted.

The risk management process can determine the quantity of the projects’ risk by studying the dimensions and elements which affect the project future performance from various aspects (opportunities and threats), and by challenging these subjects. Here, it is considered that there is no information concerning the occurrence possibility and events’ effect, or this information can be obtained by spending much time and cost. Under these circumstances each of these quantities are determined according to the project managers’ judgments and comments, hence, the calculation of these quantities based on the linguistic terms will improve the results. Since these people’s judgments bear some degrees of uncertainty and there is the probability of
error and wrong subjective judgments occurrence in them. In order to determine the risk quantity, linguistic terms must convert to fuzzy numbers. For this purpose we have used questionnaire for determining the fuzzy numbers. This questionnaire is distributed to the project experts and masters for completion till they convert the linguistic terms to fuzzy numbers in accordance with their knowledge and experiences. Some examples of guide tables of occurrence possibility and the effect quantity of the threats and opportunities (risks) are illustrated in Table 3, 4 and 5 respectively. These tables are only examples.

Table 3. Linguistic terms and fuzzy numbers related to the events’ occurrence (threats and opportunities)

<table>
<thead>
<tr>
<th>linguistic terms related to the threats</th>
<th>linguistic terms related to the opportunities</th>
<th>Fuzzy numbers</th>
</tr>
</thead>
<tbody>
<tr>
<td>The event is completely preventable.</td>
<td>There’s no probability of achieving the event and it’s achievable by no solutions.</td>
<td>[0,0,0.1,0.2]</td>
</tr>
<tr>
<td>The event occurrence probability is low.</td>
<td>The probability of achieving the event is low, but it can be achieved by hard try.</td>
<td>[0.1,0.2,0.2,0.3]</td>
</tr>
<tr>
<td>The event occurrence probability is relatively low.</td>
<td>The probability of achieving the event is relatively low, but it can be achieved by hard try.</td>
<td>[0.2,0.3,0.4,0.5]</td>
</tr>
<tr>
<td>The event occurrence probability is average but it can be prevented by extra activities</td>
<td>The probability of achieving the event is average, but it can be achieved by extra attempt.</td>
<td>[0.4,0.5,0.5,0.6]</td>
</tr>
<tr>
<td>The event occurrence probability is relatively high but it can be prevented by high cost.</td>
<td>The probability of achieving the event is relatively high.</td>
<td>[0.5,0.6,0.7,0.8]</td>
</tr>
<tr>
<td>The event occurrence probability is high; it’s possible to be prevented by other solutions which require high cost.</td>
<td>The probability of achieving the event is high.</td>
<td>[0.7,0.8,0.8,0.9]</td>
</tr>
<tr>
<td>The event cannot be prevented and certainly occurs.</td>
<td>The event is easily achievable.</td>
<td>[0.8,0.9,1,1]</td>
</tr>
</tbody>
</table>
Table 4. Linguistic terms and fuzzy numbers related to the effect quantity of the events (opportunities)

<table>
<thead>
<tr>
<th>Linguistic terms related to the cost</th>
<th>Linguistic terms related to the schedule</th>
<th>Linguistic terms related to the scope &amp; quality</th>
<th>Fuzzy numbers</th>
</tr>
</thead>
<tbody>
<tr>
<td>more than 50% decrease</td>
<td>more than 50% acceleration in moments</td>
<td>It has an indescribable and a quite desirable effect in the scope</td>
<td>[0.8,0.9,1,1]</td>
</tr>
<tr>
<td>between 40%-50% decrease</td>
<td>between 40%-50% acceleration</td>
<td>It has a high desirable effect in the scope</td>
<td>[0.7,0.8,0.8,0.9]</td>
</tr>
<tr>
<td>between 30%-40% decrease</td>
<td>between 30%-40% acceleration</td>
<td>It has a relatively high desirable effect in the scope and quality (more than average)</td>
<td>[0.5,0.6,0.7,0.8]</td>
</tr>
<tr>
<td>between 20%-30% decrease</td>
<td>between 20%-30% acceleration</td>
<td>It has a desirable effect in the scope (average)</td>
<td>[0.4,0.5,0.5,0.6]</td>
</tr>
<tr>
<td>between 10%-20% decrease</td>
<td>between 10%-20% acceleration</td>
<td>It has a relatively low desirable effect in the scope (less than average)</td>
<td>[0.2,0.3,0.4,0.5]</td>
</tr>
<tr>
<td>less than 10% decrease</td>
<td>less than 10% acceleration</td>
<td>It has a low desirable effect in the scope.</td>
<td>[0.1,0.2,0.2,0.3]</td>
</tr>
<tr>
<td>almost without cost decrease</td>
<td>almost without acceleration in schedule</td>
<td>without desirable effect</td>
<td>[0,0,0.1,0.2]</td>
</tr>
</tbody>
</table>

Table 5. Linguistic terms and fuzzy numbers related to the effect quantity of the events (threats)

<table>
<thead>
<tr>
<th>Linguistic terms related to the cost</th>
<th>Linguistic terms related to the schedule</th>
<th>Linguistic terms related to the scope &amp; quality</th>
<th>Fuzzy numbers</th>
</tr>
</thead>
<tbody>
<tr>
<td>almost without cost increase</td>
<td>almost without delay in schedule</td>
<td>It’s without desirable effect and no problem arises.</td>
<td>[0.8,0.9,1,1]</td>
</tr>
<tr>
<td>less than 10% increase</td>
<td>less than 10% delay</td>
<td>It has a low undesirable effect in the scope.</td>
<td>[0.7,0.8,0.8,0.9]</td>
</tr>
<tr>
<td>between 10%-20% increase</td>
<td>between 10%-20% delay</td>
<td>It has a relatively low undesirable effect in the scope (less than average).</td>
<td>[0.5,0.6,0.7,0.8]</td>
</tr>
<tr>
<td>between 20%-30% increase</td>
<td>between 20%-30% delay</td>
<td>It has a low undesirable effect in the scope and may be rejected by the employer(average)</td>
<td>[0.4,0.5,0.5,0.6]</td>
</tr>
</tbody>
</table>
between 30%-40% increase  |  between 30%-40% delay  |  It has a low undesirable effect in the limit and may be rejected by the employer in all probability.  |  [0.2,0.3,0.4,0.5]  
|  between 40%-50% increase  |  between 40%-50% delay  |  It has an absolutely undesirable effect in the scope and won’t be accepted by the employer.  |  [0.1,0.2,0.2,0.3]  
|  more than 50% increase  |  more than 50% delay in moments  |  It has an indescribable and absolutely undesirable effect in the scope and leads to duplication.  |  [0,0,0.1,0.2]  

\[ \hat{p}_i \text{ and } \hat{p}_j \text{ are the fuzzy numbers related to the occurrence possibility of the } i\text{th threat and the } j\text{th opportunity respectively.} \]

Also \( \hat{p}_i \) and \( \hat{p}_j \) are the fuzzy numbers related to the effect quantity of the \( i\)th threat and the \( j\)th opportunity respectively. \( n \) is the threats’ total number and \( m \) is the opportunities’ total number.

\[
\hat{p}_i = (p_{i_1}, p_{i_2}, p_{i_3}, p_{i_4}). \quad (16)
\]

\[
\hat{p}_j = (f_{j_1}, f_{j_2}, f_{j_3}, f_{j_4}). \quad (17)
\]

\[
i = 1,...,n \text{ and } j = 1,...,m
\]

\[
\hat{p}_i = (p_{i_1}, p_{i_2}, p_{i_3}, p_{i_4}). \quad (18)
\]

\[
\hat{p}_j = (f_{j_1}, f_{j_2}, f_{j_3}, f_{j_4}). \quad (19)
\]

\( \hat{R}_Q \) is the fuzzy number related to the risk quantity. \( \hat{R}_Q \) can be positive or negative. Of course it will mostly be positive, because in most cases the threats prevail over the opportunities in projects.

\[
\hat{R}_Q = \left( \sum_{i=1}^{n} \hat{p}_i \otimes \hat{p}_i \right) - \left( \sum_{j=1}^{m} \hat{p}_j \otimes \hat{p}_j \right) = [RQ_1, RQ_2, RQ_3, RQ_4]. \quad (20)
\]

\( PRI \) is a new index for evaluating the projects’ performances regarding the future opportunities and threats which will be obtained as below:

\[
\hat{PRI} = [1, 1, 1, 1] - \frac{[RQ_1, RQ_2, RQ_3, RQ_4]}{m + n} = (PRI_1, PRI_2, PRI_3, PRI_4) \quad (21)
\]
4. Ranking Fuzzy Number

The ranking of the fuzzy numbers plays an important role in decision making, optimization and forecasting. The fuzzy number related to the done estimates should be ranked before making decision in order to make an inference about the project situation. In other words, developed fuzzy estimates should be compared to the planned time and cost in order to study the existing gaps between them.

In addition to the other methods that are available for fuzzy numbers comparison (Adamo, 1980; Bortolan & Degani, 1985), a ranking method is implemented here introduced by Kumar et al. (2011).

Kumar et al. approach makes the true ranking of fuzzy numbers simply and easily possible. Kumar indicates that the ranking of the fuzzy numbers does not depend upon their height.

Fuzzy numbers $A = [a, b, c, d; w]$ and $B = [a, b, c, d; w]$ $(0 < w_i \leq 1)$ are two L-R type generalized fuzzy numbers if its membership function is given by:

$$
\mu_A(x) = \begin{cases} 
  wL(b - x / b - a), & a < x < b \\
  w, & b \leq x \leq c \\
  wL(x - c / d - c), & c < x < d 
\end{cases}
$$

Where L and R are reference functions. $A' = [a, b, c, d; 1]$ and $B' = [a, b, c, d; 1]$ are the corresponding fuzzy numbers with them and $\min(w_1, w_2) = \min(A, B)$ referring to minimum of height of the fuzzy numbers. We have:

(i) $A \geq B$ if $\mu_A(x) \geq \mu_B(x)$

(ii) $A \leq B$ if $\mu_A(x) \leq \mu_B(x)$

(iii) $A' > B'$ if $\mu_A(x) > \mu_B(x)$

In this approach the ranking of the fuzzy numbers $A = [a, b, c, d; w]$ and $B = [a, b, c, d; w]$ is as below:

$$
R(A) = \frac{w}{2} \alpha (a_i + b_i) + \frac{w}{2} (1 - \alpha) (c_i + d_i) \quad (22)
$$

$$
R(B) = \frac{w}{2} \alpha (a_i + b_i) + \frac{w}{2} (1 - \alpha) (c_i + d_i) \quad (23)
$$

Kumar et al. approach leads to certain conclusion. The operation of choosing the appropriate quantity for alpha ($\alpha$) depends upon the decision maker, his past experiences and the nature of the project.
5. Research Methodology

The considered performance indices in the EVM for estimating final results of the project is taken into account using the past performance status in time, cost or a combination of them both and does not pay attention to transformations which may occur in consequence of the conditions change of the project in the future. Due to not considering the effect possibility of different elements and future changes and transformations in the project improvement process, the authenticity and accuracy of done estimates is in question.

One of the main deficiencies in the EVM system is related to this presupposition that points out the future efficiency of the project can be forecasted by the past efficiency. Calculated operational indices in earned value, like time and cost operational indices are applied for future forecasting and will estimate the final results of the project. But there is no guarantee if the EVM basic hypothesis is true, and it’s probable that the future results will be differ from what is determined and forecasted over the previous performance. In fact, the projects’ performance is not often focused on improvement, and in the 15% progress the project performance almost never will be more than its current performance average and often will lessen (Christensen, 1994). In common methods and formulas in the EVM system, the future performance of the project is merely forecasted based on its past process, and environmental circumstances changes or other effective elements on the project performance change in future, like information availability, resources, transformation dates, material cost, being materials and etc. won’t be taken into consideration.

Considering the dominant variable conditions in projects, one can observe the role of all effective elements on the future performance in the project performance ratio determination. If the cost and schedule of project completion is under the influence of the past cost and schedule performance indices, the $\tilde{PRI}$ in accompany with the $\tilde{CPI}$ and the $\tilde{SPI}_{(v)}$ can be used for a better cost and time estimate at completion of the projects. These estimates are called adjusted schedule and cost estimate.

Here, regarding the various assumptions mentioned about calculating schedule and cost estimate at completion in section (2.2) and the risk index that we have formulated for projects, adjusted schedule and cost estimate at completion is developed as below:

I. Fuzzy adjusted cost estimate at completion (AEAC)

- In case the cost at completion is under the influence of the current cost performance indices, the fuzzy $AEAC$ (adjusted cost estimate at completion) is obtained as below:

$$
\tilde{AEAC} = AC + \frac{BAC - EV}{\tilde{CPI} \times \tilde{PRI}} = \left[ AC + \frac{BAC - EV}{\tilde{CPI_i} \times \tilde{PRI_i}}, \frac{BAC - EV}{\tilde{CPI_i} \times \tilde{PRI_i}}, \frac{AC + BAC - EV}{\tilde{CPI_i} \times \tilde{PRI_i}} \right]
$$

where $AEAC_i, AEAC_C, AEAC_{AC_i}, AEAC_{AC_C}$. (24)

- In case the cost at completion is under the influence of the current cost and schedule performance indices, we will use the following formula:
\[ \tilde{EAC} = AC + \frac{BAC - EV}{SCI * PRI} = \left[ AC + \frac{BAC - EV}{SCI * PRI} + AC + \frac{BAC - EV}{SCI * PRI} + AC + \frac{BAC - EV}{SCI * PRI} + AC + \frac{BAC - EV}{SCI * PRI} \right] = [AEAC_{i}, AEAC_{j}, AEAC_{k}, AEAC_{l}, AEAC_{m}], \quad (25) \]

II. Fuzzy adjusted schedule estimate at completion (AEAC(t))

- \( PF = 1 \): If it is supposed that the duration of the remained activities makes progress according to the plan, the fuzzy \( AEAC(t) \) (adjusted schedule estimate at completion) will be obtained as below:

\[ \tilde{EAC}_t = AD + \frac{PD - ES}{PRI} = \left[ AD + \frac{PD - ES}{PRI} + AD + \frac{PD - ES}{PRI} + AD + \frac{PD - ES}{PRI} \right] = [AEAC_{(t)}, AEAC_{(t)}, AEAC_{(t)}, AEAC_{(t)}], \quad (26) \]

- \( PF = SPI(t) \): Assuming the duration for the remained activities is influenced by the \( SPI(t) \) trend, the fuzzy \( AEAC(t) \) will be obtained as below:

\[ \tilde{EAC}_t = AD + \frac{PD - ES}{PRI} = \left[ AD + \frac{PD - ES}{PRI} + AD + \frac{PD - ES}{PRI} + AD + \frac{PD - ES}{PRI} \right] = [AEAC_{(t)}, AEAC_{(t)}, AEAC_{(t)}, AEAC_{(t)}], \quad (27) \]

- \( PF = SCI(t) \): Assuming the duration for the remained activities is influenced by the \( SCI(t) \) trend, the fuzzy \( AEAC(t) \) will be obtained as below:

\[ \tilde{EAC}_t = AD + \frac{PD - ES}{PRI} = \left[ AD + \frac{PD - ES}{PRI} + AD + \frac{PD - ES}{PRI} + AD + \frac{PD - ES}{PRI} \right] = [AEAC_{(t)}, AEAC_{(t)}, AEAC_{(t)}, AEAC_{(t)}], \quad (28) \]

The ranking of the fuzzy numbers related to the \( \tilde{EAC} \) and the \( \tilde{EAC}_{(t)} \) is calculated as below:

\[ R(\tilde{EAC}) = \frac{1}{2} \alpha (AEAC_1 + AEAC_2) + \frac{1}{2} (1 - \alpha) (AEAC_3 + AEAC_4). \quad (29) \]

\[ R(\tilde{EAC}_{(t)}) = \frac{1}{2} \alpha (AEAC_{(t)} + AEAC_{(t)}) + \frac{1}{2} (1 - \alpha) (AEAC_{(t)} + AEAC_{(t)}). \quad (30) \]

Regarding the fuzzy numbers (\( \tilde{EAC} \)) and (\( \tilde{EAC}_{(t)} \)) are normal, their height is considered 1.

The total amount of the budget at completion (BAC) of the project and its planned duration (PD) is usually determined from the beginning of the project. The project manager’s aim is that the project be completed according to the plan. The calculated forecasted cost and schedule should be compared to the planned cost and schedule. This comparison and its
related interpretations are illustrated in Tables 6 and 7:

Table 6. Interpretations related to the comparison between \( BAC \) and \( R(\bar{\text{EAC}}) \)

<table>
<thead>
<tr>
<th>Status</th>
<th>Interpretation</th>
</tr>
</thead>
<tbody>
<tr>
<td>( R(\bar{\text{EAC}}) \leq BAC )</td>
<td>The cost at completion of the project will lessen than its budgeted cost</td>
</tr>
<tr>
<td>( R(\bar{\text{EAC}}) \geq BAC )</td>
<td>The cost at completion of the project will go beyond than its budgeted cost</td>
</tr>
<tr>
<td>( R(\bar{\text{EAC}}) = BAC )</td>
<td>The cost at completion of the project will equal to its budgeted cost.</td>
</tr>
</tbody>
</table>

Table 7. Interpretations related to the comparison between \( PD \) and \( R(\bar{\text{EAC}}_{(t)}) \)

<table>
<thead>
<tr>
<th>Status</th>
<th>Interpretation</th>
</tr>
</thead>
<tbody>
<tr>
<td>( R(\bar{\text{EAC}}_{(t)}) \leq PD )</td>
<td>The time at completion of the project will lessen than its budgeted time</td>
</tr>
<tr>
<td>( R(\bar{\text{EAC}}_{(t)}) \geq PD )</td>
<td>The time at completion of the project will go beyond than its budgeted time.</td>
</tr>
<tr>
<td>( R(\bar{\text{EAC}}_{(t)}) = PD )</td>
<td>The time at completion of the project will equal to its budgeted time.</td>
</tr>
</tbody>
</table>

6. Example

In this section we give an example in order to a better illustration of our approach. The example consists of five activities which should be done over eight month. The data regarding the PV and AC are brought in Table 8 up to month four.

Table 8. AC and PV of the example

<table>
<thead>
<tr>
<th>Month</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
</tr>
</thead>
<tbody>
<tr>
<td>PV</td>
<td>1000</td>
<td>1800</td>
<td>2300</td>
<td>3100</td>
<td>3800</td>
<td>4400</td>
<td>5400</td>
<td>6000</td>
</tr>
<tr>
<td>AC</td>
<td>1200</td>
<td>2200</td>
<td>2900</td>
<td>3800</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>

The data regarding the amount of activities’ progress and the budget at completion of each activity is brought in Table 9.

Table 9. The activities’ data of the example

<table>
<thead>
<tr>
<th>activity</th>
<th>( BAC ) ($)</th>
<th>progress</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1200</td>
<td>Very high</td>
</tr>
<tr>
<td>2</td>
<td>1400</td>
<td>More than average</td>
</tr>
<tr>
<td>3</td>
<td>600</td>
<td>Less than average</td>
</tr>
<tr>
<td>4</td>
<td>800</td>
<td>Very low</td>
</tr>
<tr>
<td>5</td>
<td>2000</td>
<td>Very low</td>
</tr>
</tbody>
</table>
In Table 10 the linguistic terms regarding the progress amount of the activities 1 to 5 are converted to fuzzy numbers using Table 2. According to the $BAC$ and the completion percent of every activity, $\hat{EV}_i$ of each of the activities is obtained based on the Eqs. (1) and (2).

Table 10. Calculating the $\hat{EV}_i$ of each of the activities

<table>
<thead>
<tr>
<th>activity</th>
<th>$BAC_i$</th>
<th>Progress</th>
<th>$L_i$</th>
<th>$\hat{EV}_i$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1200</td>
<td>Very high</td>
<td>[0.8,0.9,1,1]</td>
<td>[960,1080,1200,1200]</td>
</tr>
<tr>
<td>2</td>
<td>1400</td>
<td>More than average</td>
<td>[0.5,0.6,0.7,0.8]</td>
<td>[700,840,980,1120]</td>
</tr>
<tr>
<td>3</td>
<td>600</td>
<td>Less than average</td>
<td>[0.2,0.3,0.4,0.5]</td>
<td>[120,180,240,300]</td>
</tr>
<tr>
<td>4</td>
<td>800</td>
<td>Very low</td>
<td>[0,0,0.1,0.2]</td>
<td>[0,0,80,160]</td>
</tr>
<tr>
<td>5</td>
<td>2000</td>
<td>Very low</td>
<td>[0,0,0.1,0.2]</td>
<td>[0,0,200,400]</td>
</tr>
</tbody>
</table>

Using Eq. (3) we can obtain the fuzzy earned value of the total five activities as below:

$$\hat{EV} = \sum_{i=1}^{5} \hat{EV}_i = [1780,2100,2700,3180].$$

According to the fuzzy earned value we can determine the cost and schedule indices (3100 and 3800 are the actual and planned values of month 4 respectively):

$$\hat{CPI} = \frac{\hat{EV}}{AC} = \frac{[1780,2100,2700,3180]}{[3100,3100,3100,3100]} = [0.57?0.68?0.87?1.026].$$

$$\hat{SPI} = \frac{\hat{EV}}{PV} = \frac{[1780,2100,2700,3180]}{[3800,3800,3800,3800]} = [0.47?0.55?0.71?0.84].$$

According to the presented comments in section (2.1), the calculations regarding the $ES_i$ are brought in Table 11:

Table 11. Calculations regarding $ES_i$

<table>
<thead>
<tr>
<th>$i$</th>
<th>$EV_i$</th>
<th>$N_i$</th>
<th>$EV_i - PV_{Ni}$</th>
<th>$PV_{Ni+1} - PV_{Ni}$</th>
<th>$ES_i$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1780</td>
<td>1</td>
<td>780</td>
<td>800</td>
<td>1.97</td>
</tr>
<tr>
<td>2</td>
<td>2100</td>
<td>2</td>
<td>300</td>
<td>500</td>
<td>2.6</td>
</tr>
<tr>
<td>3</td>
<td>2700</td>
<td>3</td>
<td>400</td>
<td>800</td>
<td>3.5</td>
</tr>
<tr>
<td>4</td>
<td>3180</td>
<td>4</td>
<td>80</td>
<td>700</td>
<td>4.11</td>
</tr>
</tbody>
</table>

For example, $ES_1$ is calculated as below:

$$N_1 = 1 \text{ as } PV_1 < EV_1 = 1780 < PV_2 \quad \text{thus}$$

$$ES_1 = N_1 + \left( \frac{EV_1 - PV_{Ni}}{PV_{Ni+1} - PV_{Ni}} \right) = 1 + \left( \frac{1780 - 1000}{1800 - 1000} \right) = 1.97$$
Finally the fuzzy earned schedule will be obtained as below:

\[ ES = \{ ES_1, ES_2, ES_3, ES_4 \} = [1.97, 2.6, 3.5, 4.11]. \]

According to the Eq. (10), the SPI(t) will be obtained as below:

\[ \hat{SPI}(t) = \frac{ES}{AD} = \left[ \frac{1.97}{4}, \frac{2.6}{4}, \frac{3.5}{4}, \frac{4.11}{4} \right] = [0.49, 0.65, 0.87, ?1.03]. \]

In this example we assume that three threats and two opportunities are recognized. Table 12 is related to the fuzzy data of opportunities and threats’ occurrence possibility and the amount of their effect on the time and cost of the project completion. In following tables the linguistic terms regarding the opportunities and threats are converted to the fuzzy numbers using the Table 2.

Table 12. Fuzzy numbers related to the events’ occurrence and the quantity of threats and opportunities’ effect

<table>
<thead>
<tr>
<th>Threats</th>
<th>Pi</th>
<th>Fi</th>
<th>Pi* Fi</th>
</tr>
</thead>
<tbody>
<tr>
<td>T1</td>
<td>[0.4,0.5,0.5,0.6]</td>
<td>[0.4,0.5,0.5,0.6]</td>
<td>[0.16,0.25,0.25,0.36]</td>
</tr>
<tr>
<td>T2</td>
<td>[0.5,0.6,0.7,0.8]</td>
<td>[0.5,0.6,0.7,0.8]</td>
<td>[0.25,0.36,0.49,0.64]</td>
</tr>
<tr>
<td>T3</td>
<td>[0.2,0.3,0.4,0.5]</td>
<td>[0.4,0.5,0.5,0.6]</td>
<td>[0.08,0.15,0.20,0.30]</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Opportunities</th>
<th>Pj</th>
<th>Fj</th>
<th>Pj*Fj</th>
</tr>
</thead>
<tbody>
<tr>
<td>O1</td>
<td>[0.5,0.6,0.7,0.8]</td>
<td>[0.7,0.8,0.8,0.9]</td>
<td>[0.35,0.48,0.56,0.72]</td>
</tr>
<tr>
<td>O2</td>
<td>[0.2,0.3,0.4,0.5]</td>
<td>[0.2,0.3,0.4,0.5]</td>
<td>[0.04,0.09,0.16,0.25]</td>
</tr>
</tbody>
</table>

The whole project risk quantity (\( \hat{RQ} \)) based on the Eq. (20) will be obtained as below:

\[ \hat{RQ} = ([0.16,0.25,0.25,0.36] + ... - [0.04,0.09,0.16,0.25]) = [0.10,0.19,0.22,0.33]. \]

The risk ratio based on the Eq. (21) is calculated as below:

\[ \hat{PRI} = \frac{\frac{0.33}{2} [0.22\% 0.19\% 0.10\%]}{2+3} = 0.934, 0.956, 0.962, 0.98. \]

So far, we have achieved the earned value management cost and schedule indices and risk ratio. Now we should estimate the adjusted schedule and cost at completion of the project. (Here it is assumed that the cost at completion of the project and the remained activities duration is influenced by the CPI and the SPI(t) trend respectively):

\[ \hat{AEAC} = \left[ \frac{3800}{1.026 * 0.98}, \frac{6000 - 3180}{0.87 * 0.962}, \frac{6000 - 2700}{0.8 * 0.956}, \frac{6000 - 2100}{0.68 * 0.956}, \frac{6000 - 1780}{0.57 * 0.934} \right] = [6605, 743, 9800, 11727]. \]
Having the fuzzy quantities, one can calculate the ranked adjusted schedule and cost estimate. In this example, the alpha is considered (0.7).

\[
\begin{align*}
R(\overline{EAC}_0) &= \frac{1}{2} \left( 0.7(6605 + 7743) + \frac{1}{2}(1 - 0.7)(9800 + 11727) \right) = 8252.22 \\
R(\overline{EAC}_0) &= \frac{1}{2} \left( 0.7(7.85 + 9.37) + \frac{1}{2}(1 - 0.7)(12.7 + 17.18) \right) = 10.5
\end{align*}
\]

The results indicate that if the operation continues in this way, the cost and schedule at completion of the project will increase.

7. Conclusion

The earned value management in calculating the schedule and cost performance indices assumes that the future performance of the project is a function of its past performance. Despite opportunities and threats which have surrounded a lot of projects in current environment, this assumption cannot be true. However, the researches carried out in the earned value management area are firmly based on this assumption, while the project future performance might not be compatible with its past performance under no circumstances. In this research we have presented a model that assists projects’ managers estimate the future status of the project with more certainty. To this end, recognizing the threats and opportunities that projects may face in the future we calculate the risky ratio for combining with the project performance indices. This risk ratio, in fact, adjusts the earned value management indices. Furthermore, we have calculated these indices and risk ratio based on the fuzzy technique, because they are on the basis of various peoples’ comments and judgments. We believe that by doing both calculations we can estimate the projects’ schedule and cost with more certainty.

This model encourages us to evaluate its application in real environment in future researches. Furthermore, we can develop a more comprehensive model out of the current model in order to a better evaluation of projects future performance.

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References


