Optimization of Empty Container Repositioning in Liner Shipping

Hüseyin Gençer (Corresponding author)
Piri Reis University, Turkey
E-mail: e.huseyin.gencer@gmail.com

M. Hulusi Demir
Netkent Mediterranean Research and Science University, North Cyprus
E-mail: hulusi.demir@gmail.com

Received: Dec. 5, 2019   Accepted: Jan. 2, 2020   Published: Jan. 22, 2020
doi:10.5296/bmh.v8i1.16327     URL: http://dx.doi.org/10.5296/bmh.v8i1.16327

Abstract
Empty container repositioning (ECR), which arises due to imbalances in world trade, causes extra costs for the container liner carrier companies. Therefore, one of the main objectives of all liner carriers is to reduce ECR costs. Since ECR decisions involve too many parameters, constraints and variables, the plans based on real-life experiences cannot be effective and are very costly. For this purpose, this study introduces two mathematical programming models in order to make ECR plans faster, more efficient and at the lowest cost. The first mathematical programming model developed in this study is a mixed-integer linear programming (MILP) model and the second mathematical programming model is a scenario-based stochastic programming (SP) model, which minimize the total ECR costs. Unlike the deterministic model, the SP model takes into account the uncertainty in container demand. Both models have been tested with real data taken from a liner carrier company. The numerical results showed that, in a reasonable computational time, both models provide better results than real-life applications of the liner carrier company.

Keywords: empty container repositioning, mathematical programming, container shipping
1. Introduction

A large part of international trade is carried out by maritime transportation. Because container ensures safe, comfortable and intermodal transportation opportunities, container transportation is one of the most preferred type of cargo transportation for long distances in international trade and maritime transportation. A very large proportion of the world container fleet is used in liner shipping which is the backbone of container shipping. The imbalances in the worldwide trade affect the container traffic directly and cause imbalances in the number of containers in many locations. In other words, export dominant locations have deficits in terms of number of containers and import dominant locations do surpluses. As a result, surplus containers should be repositioned as empty to the locations where they are required.

Figure 1 shows the estimated containerized cargo flow on major trade lines in 2015. As can be seen, there are many exports from Asia to Europe. Conversely, the imports to Asia from Europe are not even half of this rate. In this regard, the following comment can be made: Approximately one of the two 20dv full containers going from Asia to Europe, will not be used after the containers have been unstuffed. On the other hand, empty containers are required in Asia to meet the demand for export cargos. Hence, empty containers, which are not required in Europe, are transported back to the locations in Asia.

![Figure 1. Containerized cargo flows on major trade routes in 2015 (in million TEUs) (Source: UNCTAD, Lloyd’s List, Statista)](image)

As can be seen in Figure 1, a similar imbalance exists in the containerized trade between Europe and North America, and Eastern Asia and North America. A similar trade imbalance on the main routes is still present today. Empty containers that are held in storage, handled and repositioned due to trade imbalances lead to extra costs for the liner carriers. More clearly, while the liner carriers make money by carrying full containers, they spend money for the repositioning of empty containers. According to many experts, the total empty container
repositioning (ECR) costs exceed 20 billion dollars annually. Therefore, one of the main objectives of all liner carrier companies is to reduce the repositioning costs of empty containers.

The decision-making process for ECR is a very complex structure involving many factors which are in continuous contact with each other. Thus, decisions taken based on past and intuitive experiences cannot be very efficient. For that purpose, this study introduces two mathematical programming models in order to make ECR plans faster, more efficient and at the lowest cost. The first mathematical programming model developed in this study is a mixed-integer linear programming (MILP) model and the second mathematical programming model is a scenario-based stochastic programming (SP) model, which minimize the ECR costs. Besides the various objectives and constraints in the real-life applications, SP model also takes into account the uncertainty of container demand in the locations.

2. Literature Review

The issue of container imbalances has been a point of interest in the academic world as well. Many researchers have addressed this issue from different aspects. Most of the studies in the literature focus on the optimization of ECR resulting from container imbalances and aim to find the optimal empty container distribution plan. The vast majority of optimization models developed in the literature are Linear Programming (LP) models and variations. Heuristic methods have also been developed in some studies especially for the solution of large scale problems where there are too many variables and constraints.

The studies on ECR started in the early 1970s (Potts, 1970; White, 1972) and increased especially in the 2000s. Most of the studies examined ECR between the seaports (Shen and Khoong, 1995; Choong, 2002; Shintani et al., 2007; Feng and Chang, 2008; Dong and Song, 2012; Zhang and Fachanha, 2014; Huang, 2015, Akyüz and Lee, 2016). Some researchers studied the ECR problem for inland transportation (Wang and Wang, 2007; Dang, 2012; Furio et al., 2013; Xie, 2017). All of these studies approached the ECR problem from a deterministic point of view. The objective function in most of these studies was to minimize the ECR costs, including transportation, handling and storage costs. Only a few studies considered other costs such as container leasing and repair costs (Moon et al. 2013; Hjortnaes et al., 2017)

Although many studies in the literature dealt with ECR in a deterministic way, there are many uncertainties in real life applications. As the parties in container shipping have different purposes; vessel arrivals, vessel space and weight capacities, customer demands can change at any time. Especially, it is crucial to meet customer demand on time in liner shipping. Therefore, in some studies, the uncertainties that may arise in ECR real-life operations were taken into consideration (Lai et al., 1995; Cheung and Chen, 1998; Li et al., 2004; Song, 2006; Lam et al., 2007; Erera et al., 2009; Francesco et al., 2009; Long et al., 2012; Mittal et al., 2013; Wong et al., 2015). Similar to deterministic models, the objective function in all these studies was to minimize the ECR costs. Wang and Tang (2012) introduced a chance-constraint programming model which maximizes total profit taking into account empty and full containers.
Some researches have addressed the ECR problem with the transportation of full containers (Bandeira et al., 2009; Dong and Song, 2009; Wang and Tang, 2010). This approach is more robust considering that empty and full containers are transported together and directly affects the allocation of empty containers on vessels. In other words, it is more realistic for industry applications.

Only a few of the studies in the literature, in which both deterministic and stochastic models were developed, took into account different types of containers. In these studies, the incremental stock situation in terminals or depots has not been addressed at all.

Some studies highlighted the importance and advantages of collaboration by examining container sharing practices between liner carriers (Le, 2003; Song, 2007; Song and Carter, 2009). These studies have shown that container sharing practices can provide liner carriers with significant cost savings.

Moon et al. (2013) studied the repositioning costs of foldable containers. Their study has shown that repositioning of the foldable containers is less costly than the standard containers. Myung and Moon (2014) researched the repositioning of both standard and foldable empty containers. The authors considered the ECR problem as a minimum cost network problem. They developed a single commodity minimum cost network flow model which finds the optimal solution. Shintani et al. (2019) dealt with the transportation of full and empty containers. The authors also took into account the combinable containers and developed a minimum cost multi-commodity network flow model. The study showed that mixed use of both standard and combinable containers can provide significant cost savings.

Kuzmicz and Pesch (2019) studied the ECR considering the impact of Chinese One Belt One Road (OBOR) initiative on Euroasian intermodal transportation. The authors discussed the approaches to ECR and summarized prominent studies in this field.

The ECR issue will continue due to imbalances in world trade. Therefore, although there are a large number of studies on the ECR problem in the literature, studies focusing on this field are still being conducted. For the researches on ECR and management, studies of Theofanis and Boile (2008), Braekers et al. (2011), Lee and Song (2017), Gençer and Demir (2019) can be referred as detailed literature reviews.

3. Mathematical Programming Models

Almost all parties, especially the liner carriers in container shipping industry struggle with the high costs of ECR. Therefore, making efficient plans to reduce these costs is crucial for the container shipping companies. Objective of this study is to develop mathematical programming models that minimize the total costs of ECR in order to make more efficient and faster plans. The models developed in this study are described below.

3.1 Mixed Integer Linear Programming Model

Container shipping is a very suitable field for the applications of variations of LP models. Because container shipping is a cellular type of transportation and large number of customers are serviced on the same vessel, it is an extremely attractive issue not only for the operations
approaches, but also for the pricing and costing researches. As the deterministic mathematical programming model developed in this section involves both binary and integer variables, it is a MILP model. The description, assumptions and deterministic solution model of the ECR problem are described below.

Problem Description:

The container liner carrier company, whose operations and data were examined in the development of mathematical programming models, serves its customers by using third party transportation providers. Other than carrying full containers, the liner carrier also repositions its empty containers via third party vessels, trucks and trains in order to meet the demand of export dominant locations. In particular, the liner carrier company is strictly dependent on the schedule and capacity allocation of other liner carriers' vessels. Therefore, it needs to make detailed plans a few weeks in advance to cover the demand and minimize the total ECR costs.

The liner carrier company provides weekly container shipping services to its customers and has some imbalances in some locations in a specific region. Accordingly, the excess empty containers in the import dominant locations must be sent to the export dominant locations to cover the container demand. Because the vessel transportation service in the region lasts four weeks at most, a four-week ECR plan is needed. For this purpose, a MILP model was developed which considers the transportation and storage costs of empty containers.

Although there are more locations within the service region of the liner carrier company, it is not necessary to include these locations in the data set of the problems, as there are only one transportation option for repositioning excess empty containers to the outside of the region. Therefore, these locations are not considered in the weekly data sets to reduce the number of parameters and decision variables in the problems, depending on the requirement for empty containers in the export dominant locations. Only three types of containers are taken into consideration since the container types that the liner carrier company carries the most in this region are 20dv, 40dv and 40HC. In the MILP model, ECR costs do not include the terminal handling cost (THC) in the locations, but the transportation costs between the locations. Since the excess containers from import dominant locations have to be repositioned to any export dominant locations at any time and these export dominant locations will receive empty containers at any time, the THC in the locations would occur at all events. This is why the THC in the locations are not included in the LP model. If the THC were also taken into account within the ECR costs in the MILP model, it would cause containers to be accumulated in the surplus locations, preventing the empty containers from being sent from these locations due to high THC. Other assumptions, sets, parameters and decision variables for the MILP model are as follows:

- Import and export numbers of container types in each location are known for the four-week period.
- All the surplus containers are available in the locations and ready to be repositioned.
- The weekly demand of each export dominant location must be covered.
- Vessel arrival times and capacities are known for each week.
- Empty containers arrive in the deficit locations at the beginning of the week and free storage times at terminals are taken into account.
- Apart from the empty containers remaining for the next week, other empty containers will be used within that week and no storage costs will incur for those empty containers.

**Sets:**

- \( E = \{1 \ldots |E|\} \) set of the surplus locations – indexed by \( e \).
- \( D = \{1 \ldots |D|\} \) set of the deficit locations – indexed by \( d \).
- \( I = \{1 \ldots |I|\} \) set of the transportation connections between the locations – indexed by \( i \).
- \( T = \{1 \ldots |T|\} \) set of the container types – indexed by \( t \).
- \( W = \{1 \ldots |W|\} \) set of the weeks – indexed by \( w \).
- \( C = \{1 \ldots |C|\} \) set of the stock levels – indexed by \( c \).

**Parameters:**

\[
\begin{align*}
C_{edti} & \quad \text{transportation cost of an empty container type of } c \text{ from location } e \text{ to } d \text{ via } i \in \{t, e, d, i\} \in \{t, e, d, i\} \in T \\
E_{edt} & \quad \text{weekly storage cost of an empty container type of } c \text{ at location } e \text{ for stock level } t \in \{e, t, e, t\} \in \{e, t, e, t\} \in \{e, t, e, t\} \in C \\
DC_{edt} & \quad \text{weekly storage cost of an empty container type of } c \text{ at location } e \text{ for stock level } t \in \{d, D, c\} \in \{d, D, c\} \in \{d, D, c\} \in C \\
E_{etw} & \quad \text{number of containers of type } c \text{ that can be sent out of location } e \text{ in week } w \in \{e, t, w\} \in \{e, t, w\} \in \{e, t, w\} \in W \\
Y_{dtw} & \quad \text{number of containers of type } c \text{ that are needed at location } d \text{ in week } w \in \{d, t, w\} \in \{d, t, w\} \in \{d, t, w\} \in W \\
Max_{et} & \quad \text{total capacity of the connection } i \text{ from location } e \text{ to } d \text{ via } i \in \{e, d, i\} \in \{e, d, i\} \in \{e, d, i\} \in I \\
B_{t} & \quad \text{Size of the container type } c \text{ in terms of TEU, } c \in C \\
T_{edtiw} & \quad \text{transportation time from location } e \text{ to } d \text{ via } i \text{ in week } w \in \{e, d, i, w\} \in \{e, d, i, w\} \in \{e, d, i, w\} \in W \\
SQ_{etw} & \quad \text{total stock quantity in terms of TEU at location } e \text{ for stock level } t \text{ in week } w \in \{e, w, t\} \in \{e, w, t\} \in \{e, w, t\} \in C \\
SQ_{dtw} & \quad \text{total stock quantity in terms of TEU at location } d \text{ for stock level } t \text{ in week } w \in \{d, w, t\} \in \{d, w, t\} \in \{d, w, t\} \in C \\
M & \quad \text{a very large integer number}
\end{align*}
\]
Decision variables:

\[ \begin{align*} 
Q_{s,d,t,w} & \quad \text{number of containers of type } t \text{ sent from location } s \text{ to } d \text{ via } t \text{ in week } w, \; \forall s,d,t,w \in W \\
G_{s,d,t,w} & \quad \text{number of containers of type } t \text{ coming from location } s \text{ to } d \text{ via } t \text{ in week } w, \; \forall s,d,t,w \in W \\
R_{s,w} & \quad \text{number of containers of type } t \text{ remaining at location } s \text{ in week } w, \; \forall s,t,w \in W \\
K_{s,w} & \quad \text{number of containers of type } t \text{ remaining at location } s \text{ in week } w, \; \forall s,t,w \in W \\
T_{v,w} & \quad \text{if function } v \text{ has the stock level } v \text{ in week } w, \; \forall v,w \in V, \; v \in C \\
I_{v,w} & \quad \text{if function } v \text{ has the stock level } v \text{ in week } w, \; \forall v,w \in V, \; v \in C \\
\end{align*} \]

Objective function:

\[ \sum_{s} \sum_{w} \sum_{e} \sum_{g} R_{s,w} \cdot BC_{s,e} \cdot T_{e,w} + \sum_{d} K_{d,w} \cdot DC_{d,w} \cdot I_{d,w} + \sum_{s} \sum_{d} \sum_{l} G_{s,d,t} \cdot C_{s,d,t} \]

Constraints:

\[ O_{s,d,t,w} = \begin{cases} G_{s,d,w} & \forall s,d,t,w \\
\sum_{t} G_{s,d,t} \geq Y_{d,w} & \forall d,w \end{cases} \]

\[ \begin{align*} 
\sum_{e} \sum_{t} G_{s,d,t} & \geq Y_{d,w} \quad \forall d,w \in \{2, ..., |W|\} \\
K_{d,w} &= \sum_{e} \sum_{t} G_{s,d,t} - Y_{d,w} \quad \forall d,w \in \{2, ..., |W|\} \\
K_{d,w} &= \sum_{e} \sum_{t} G_{s,d,t} - Y_{d,w} + K_{d,w-1} \quad \forall d,t,w \in \{2, ..., |W|\} \\
\sum_{g} G_{s,d,t} \cdot B_{g} & \leq K_{d,w} \quad \forall s,d,t,w \\
\sum_{s} \sum_{t} O_{s,d,t} & \leq F_{s,d} \quad \forall s,t \\
\sum_{s} \sum_{t} O_{s,d,t} & \leq F_{s,d} + R_{s,d,w-1} \quad \forall s,t,w \in \{2, ..., |W|\} \\
\end{align*} \]
The objective function minimizes the total ECR costs in the four-week period. Constraint set (1) indicates when the empty containers’ type of t that are sent from location e via connection i will be in location d. Constraints (2) and (3) ensure that all the weekly demands for each type of container t in each deficit location d must be covered. Constraint sets (4) and (5) show the number of empty containers for each type of t left in each deficit location in each week. Constraint (6) guarantees that the total number of TEUs of empty containers to be transported from surplus location e to deficit location d cannot exceed the capacity of connection i. Constraints (7) and (8) ensure that each surplus location e cannot send more empty containers type of t than existing containers in this location in each week. Constraint sets (9) and (10) show the number of empty containers for each type of t left in each surplus location e in each week. Constraint (11) shows the weekly stock level in each deficit location d in terms of TEU. Constraint set (12) indicates that each deficit location d will have one stock level. Constraint (13) demonstrates the weekly stock level in each surplus location e in terms of TEU. Constraint (14) ensures that each surplus location e will have one stock level. Constraints (15) and (16) represent the range of decision variables.

### 3.2 Stochastic Programming Model

One of the most challenging situations for the shipping companies is the uncertainty in container demands. A sudden increase in container demand can cause all plans to be overturned, as well as unanticipated costs. Moreover, the failure to meet container demands can also lead to customer losses. Therefore, the uncertainty or change in the container demands should be taken into consideration when making ECR plans. When making the ECR plans, it is taken into account that the booking numbers in the first week, i.e. the container demands, are exactly known. For this purpose, three different scenarios were taken into
consideration for weekly demand for each container type in the deficit locations after the first week in line with the needs of the liner carrier company whose data used in this study. The probabilities of the scenarios were generated according to the discrete distribution of past data as well as the opinions of company managers. The managers’ opinions were consulted about the seasonality effect and the possibility of realization of the bookings that set up container demands in that week. The discrete distribution was readily available from the business intelligence program that the liner carrier company uses, and was determined to be proportional to the frequency percentages of the container demands. Other assumptions are as the same as in the deterministic model.

**Sets:**

\[ E = \{1, \ldots, |E|\} \] set of the surplus locations- indexed by e.
\[ D = \{1, \ldots, |D|\} \] set of the deficit locations – indexed by d.
\[ I = \{1, \ldots, |I|\} \] set of the transportation connections between the locations – indexed by i.
\[ T = \{1, \ldots, |T|\} \] set of the container types – indexed by t.
\[ W = \{1, \ldots, |W|\} \] set of the weeks – indexed by w.
\[ C = \{1, \ldots, |C|\} \] set of the inventory levels – indexed by c.
\[ S = \{1, \ldots, |S|\} \] set of the scenarios – indexed by s.

**Parameters:**

\[ C_{ext} \] transportation cost of an empty container type of t from location e to d via \( I \), \( e \in E, d \in D, i \in I, t \in T \)
\[ BC_{ext} \] weekly storage cost of an empty container type of t at location e, \( e \in E, d \in D, t \in T, c \in C \)
\[ BC_{est} \] weekly storage cost of an empty container type of t at location e, \( d \in D, t \in T, c \in C \)
\[ F_{ext} \] number of containers of type t that can be sent out of location e in week w, \( e \in E, t \in T, w \in W \)
\[ Y_{ext} \] number of containers of type t that are needed at location d in week w for scenario s, \( d \in D, t \in T, w \in W, s \in S \)
\[ K \] total capacity of the connection i from location e to d, \( e \in E, d \in D, i \in I \)
\[ B_t \] Size of the container type t in terms of TEU, \( t \in T \)
\[ E_{ext} \] transportation time from location e to d via \( I \) in week w, \( e \in E, d \in D, i \in I, w \in W \)
\[ S(t) \] total stock quantity in terms of TEU at location e for stock level c in week w for scenario s, \( e \in E, w \in W, c \in C, s \in S \)
Decision variables:

- \( g_{\text{sent}} \) number of containers of type \( t \) sent from location \( e \) to \( d \) via \( t \) in week \( w \) for scenario \( s \), \( e \in E, d \in D, t \in T, w \in W, s \in S \)
- \( g_{\text{rec}} \) number of containers of type \( t \) coming from location \( e \) to \( d \) via \( t \) in week \( w \) for scenario \( s \), \( e \in E, d \in D, t \in T, w \in W, s \in S \)
- \( R_{\text{ent}} \) number of containers of type \( t \) remaining at location \( e \) in week \( w \) for scenario \( s \), \( e \in E, t \in T, w \in W, s \in S \)
- \( R_{\text{ent}} \) number of containers of type \( t \) remaining at location \( e \) in week \( w \) for scenario \( s \), \( e \in E, t \in T, w \in W, s \in S \)

Objective function:

\[
\sum_s \sum_t \sum_e \sum_w \sum_v \left( R_{\text{ent}} \times IC_{\text{ent}} + \sum_h \left( K_{\text{d}} \times DC_{\text{d}} + IC_{\text{d}} \right) \right)
\]

Constraints:

1. \( g_{\text{sent}} = g_{\text{sent}}(w+t+s) \) \( \forall e, d, t, w, s \) \( \quad (1) \)
2. \( \sum_t g_{\text{sent}} \geq Y_{\text{d}} \) \( \forall d, s \) \( \quad (2) \)
3. \( \sum_t g_{\text{sent}} + K_{\text{d}} \geq Y_{\text{d}} \) \( \forall d, t, w \in \{2, ..., W\}, s \) \( \quad (3) \)
4. \( H_{\text{d}} = \sum_t g_{\text{ent}} - Y_{\text{d}} \) \( \forall d, t \) \( \quad (4) \)
The objective function minimizes the total ECR costs for all scenarios in the four-week period. Constraint set (1) indicates for each scenario s when the empty containers’ type of t that are sent from location e via connection i will be in location d. Constraints (2) and (3) ensure that all the weekly demands of each type of container t for each scenario s in each deficit location d must be covered. Constraint sets (4) and (5) show the number of empty containers for each type of t left in deficit location d in each week for scenario s. Constraint (6) guarantees that the total number of TEUs of empty containers in each week to be transported from surplus location e to deficit location d for each scenario s cannot exceed the capacity of connection i. Constraints (7) and (8) ensure that each surplus location e cannot send more empty containers type of t for each scenario s than existing containers in this location in each week. Constraint (9) shows the weekly stock level for each scenario s in each deficit location d in terms of TEU. Constraint (10) ensures that each deficit location d will have one stock level for each scenario s. Constraint sets (11) and (12) show the number of empty containers for each type of t left in surplus location e in each week for scenario s. Constraint (13) show the weekly stock level for each scenario s in each surplus location e in
terms of TEU. Constraint set (14) indicates that each surplus location e will have one stock level for each scenario s. Constraints (15) and (16) represent the range of decision variables.

4. Numerical Experiments

The models were tested with the real data taken from the liner carrier company. Because the total of 8 weeks from the first and last weeks in a one-year data of the liner carrier company were not clear, they have not been used for the numerical experiment. Numerical results for the models are shown separately below.

Experimental studies were carried out to measure the applicability and effectiveness of the models. The proposed model was solved with the 44 weeks of data of the liner carrier company. The solutions were compared with the ECR plans made and implemented by the liner carrier company in terms of cost efficiency and solution time. The experimental studies are carried out via IBM ILOG CPLEX 12.6 on a computer of Intel(R) Core(TM) i5-4210U CPU 2.40 Ghz processor - 4 GB RAM.

4.1 Numerical Results for the Mixed-Integer Linear Programming Model

Table 1 shows the solution times of the data sets. Each problem data has 3884 constraints, 190 binary and 5685 integer variables out of 5876 variables. As can be seen, the longest solution time is 6,61 CPU seconds. So the MILP model solves the real-life problems very quickly. The comparison of the results of the MILP with real life applications of the liner carrier company is given in Table 2.

<table>
<thead>
<tr>
<th>Data set</th>
<th>CPU</th>
<th>Real time</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>6.50 s</td>
<td>6.03 s</td>
</tr>
<tr>
<td>2</td>
<td>3.83 s</td>
<td>4.36 s</td>
</tr>
<tr>
<td>3</td>
<td>4.77 s</td>
<td>5.50 s</td>
</tr>
<tr>
<td>4</td>
<td>4.19 s</td>
<td>4.79 s</td>
</tr>
<tr>
<td>5</td>
<td>3.42 s</td>
<td>3.98 s</td>
</tr>
<tr>
<td>6</td>
<td>6.17 s</td>
<td>6.81 s</td>
</tr>
<tr>
<td>7</td>
<td>5.52 s</td>
<td>6.10 s</td>
</tr>
<tr>
<td>8</td>
<td>3.94 s</td>
<td>4.47 s</td>
</tr>
<tr>
<td>9</td>
<td>3.26 s</td>
<td>3.88 s</td>
</tr>
<tr>
<td>10</td>
<td>4.56 s</td>
<td>5.19 s</td>
</tr>
<tr>
<td>11</td>
<td>6.61 s</td>
<td>7.24 s</td>
</tr>
</tbody>
</table>

As Table 2 shows, MILP model solutions provide up to 8.76 % cost savings over real life applications. Although the model's improvement rate is low, it can be said that it has improved the decisions significantly taking into account cost savings of thousands of dollars.
Table 2. Comparison of the MILP model solution with the real-life applications

<table>
<thead>
<tr>
<th>Data set</th>
<th>MILP Model (Total ECR costs in USD)</th>
<th>Liner carrier's application (Total ECR costs in USD)</th>
<th>Improvement</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>189237</td>
<td>207425</td>
<td>8.76 %</td>
</tr>
<tr>
<td>2</td>
<td>177565</td>
<td>183294</td>
<td>3.12 %</td>
</tr>
<tr>
<td>3</td>
<td>167738</td>
<td>171190</td>
<td>2.01 %</td>
</tr>
<tr>
<td>4</td>
<td>185412</td>
<td>194853</td>
<td>4.84 %</td>
</tr>
<tr>
<td>5</td>
<td>186290</td>
<td>196084</td>
<td>4.99 %</td>
</tr>
<tr>
<td>6</td>
<td>179424</td>
<td>185412</td>
<td>3.22 %</td>
</tr>
<tr>
<td>7</td>
<td>194325</td>
<td>202770</td>
<td>4.16 %</td>
</tr>
<tr>
<td>8</td>
<td>174606</td>
<td>184920</td>
<td>5.57 %</td>
</tr>
<tr>
<td>9</td>
<td>185553</td>
<td>198366</td>
<td>6.45 %</td>
</tr>
<tr>
<td>10</td>
<td>168506</td>
<td>177934</td>
<td>5.29 %</td>
</tr>
<tr>
<td>11</td>
<td>212169</td>
<td>221147</td>
<td>4.05 %</td>
</tr>
</tbody>
</table>

4.2 Numerical Results for the Stochastic Programming Model

As mentioned earlier, the SP model considers three different scenarios for weekly demand of each deficit location and three container types. The demand quantities and probabilities in these three scenarios were taken from the liner carrier's business intelligence program. The data except for the demands are exactly the same as the data used in the MILP model.

Table 3 shows the solution times of the data sets. Each problem data has 11458 constraints, 570 binary and 17055 integer variables out of 17626 variables. As can be seen, the increase in the number of scenarios in the demands has also enlarged the problem size and extended the solution time significantly.

Depending on the difficulty of the problems in the data sets, some problems have exact solutions within a few minutes, while the computational time of some exact solutions have exceeded 14 hours in real time.

Table 3. Computational solution time of the data sets with SP model

<table>
<thead>
<tr>
<th>Data set</th>
<th>CPU</th>
<th>Real time</th>
<th>Optimality gap</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>46.50 seconds</td>
<td>48.04 minutes</td>
<td>Exact solution</td>
</tr>
<tr>
<td>2</td>
<td>2957.51 seconds</td>
<td>49.28 minutes</td>
<td>Exact solution</td>
</tr>
<tr>
<td>3</td>
<td>73.01 seconds</td>
<td>1.23 minutes</td>
<td>0.10%</td>
</tr>
<tr>
<td>4</td>
<td>2515.76 seconds</td>
<td>40.33 minutes</td>
<td>0.10%</td>
</tr>
<tr>
<td>5</td>
<td>481.11 seconds</td>
<td>9.33 minutes</td>
<td>0.10%</td>
</tr>
<tr>
<td>6</td>
<td>1040.59 seconds</td>
<td>18.41 minutes</td>
<td>0.10%</td>
</tr>
<tr>
<td>7</td>
<td>355.34 seconds</td>
<td>8.23 minutes</td>
<td>0.10%</td>
</tr>
<tr>
<td>8</td>
<td>119.27 seconds</td>
<td>2.01 minutes</td>
<td>Exact solution</td>
</tr>
<tr>
<td>9</td>
<td>100.08 seconds</td>
<td>1.41 minutes</td>
<td>Exact solution</td>
</tr>
<tr>
<td>10</td>
<td>197.25 seconds</td>
<td>4.56 minutes</td>
<td>0.10%</td>
</tr>
<tr>
<td>11</td>
<td>121.98 seconds</td>
<td>2.3 minutes</td>
<td>0.10%</td>
</tr>
</tbody>
</table>
When these solution results were examined, it was noticed that there was no significant improvement in the objective function after one hour. For this purpose, a 0.1% gap is set for the data sets that take more than one hour for the optimal solution. This 0.1% is the optimality gap or the distance from the possible optimal solution to the problem. This indicates that there might be a better result of 0.1%. Accordingly, the optimal solution for any data set which have a 0.1% gap will not be less than a few hundred dollars of the total cost. As Table 4 shows, all of the SP model solutions give essentially better results than real life applications of the liner carrier company. Namely, the liner carrier company can save thousands of dollars in a 4-week period. Considering that the model was used in other regions where the company served, the annual savings would be hundreds of thousands of dollars.

Table 4. Comparison of the SP model solution with the real-life applications

<table>
<thead>
<tr>
<th>Data set</th>
<th>SP Model (Total ECR costs in USD)</th>
<th>Liner carrier's application (Total ECR costs in USD)</th>
<th>Improvement</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>201203,9</td>
<td>207425</td>
<td>2.99%</td>
</tr>
<tr>
<td>2</td>
<td>181706,4</td>
<td>183294</td>
<td>0.86%</td>
</tr>
<tr>
<td>3</td>
<td>167911,2</td>
<td>171190</td>
<td>1.91%</td>
</tr>
<tr>
<td>4</td>
<td>192239,4</td>
<td>194853</td>
<td>1.34%</td>
</tr>
<tr>
<td>5</td>
<td>192454,1</td>
<td>196084</td>
<td>1.85%</td>
</tr>
<tr>
<td>6</td>
<td>183921</td>
<td>185412</td>
<td>0.8%</td>
</tr>
<tr>
<td>7</td>
<td>197070</td>
<td>202770</td>
<td>2.81%</td>
</tr>
<tr>
<td>8</td>
<td>182803,5</td>
<td>184920</td>
<td>1.14%</td>
</tr>
<tr>
<td>9</td>
<td>195223,2</td>
<td>198366</td>
<td>1.58%</td>
</tr>
<tr>
<td>10</td>
<td>172884,2</td>
<td>177934</td>
<td>2.83%</td>
</tr>
<tr>
<td>11</td>
<td>216860,6</td>
<td>221147</td>
<td>1.93%</td>
</tr>
</tbody>
</table>

5. Conclusion

One of the initial objectives of container liner carriers is to minimize the total ECR costs. The ECR problem has also been studied in the academic world and various optimization models have been developed. Most of these models are LP and variations, and SP models.

This study introduced two optimization models which minimize the total ECR costs in container liner shipping. The first model is a MILP model that minimizes the ECR cost assuming that the weekly container demands are exactly known, as in other parameters and variables. The second model is a SP model which takes into account the uncertainties in container demands. The models are similar to many mathematical programming models in the literature, but they also include different types of containers that have been taken into consideration only in a few articles. Moreover, unlike other studies in the literature, the models also take into account the incremental stock level applied in many container terminals in real life.

The MILP model introduced in the study gave optimum solutions in a very short computational time and provided cost savings up to 8.76% compared to real-life practices of the liner carrier. The SP model presented in the study also gave much more robust results.
when there are uncertainties in some parameters and decision variables. The SP model includes three possible scenarios for the container demands in the deficit locations in the second, third and fourth week. This has enlarged the problem size and extended the computational solution times. Nevertheless, the solutions of the SP model showed better results for the real-life applications within a reasonable computational time and provided cost savings up to 2.99%.

Since the ECR operations in container shipping are usually the same, the models can be tested and applied with different data. Namely, all container liner carriers for their ECR operations can use the models presented in this study.

Container shipping is a very attractive research area for the implementation of operations research techniques. In this respect, the models developed in this study can be modified slightly and formulated as other modeling methods such as dynamic programming so that the ECR can be approached with different perspectives. Moreover, the models can be transformed into vessel routing problems by taking into account full containers in order to make better decisions in the whole container shipping management.

Acknowledgments

The authors would like to thank the anonymous reviewers for their helpful suggestions to improve the paper.

References


Dong, J., & Song, D. (2009). Container fleet sizing and empty repositioning in liner shipping


**Copyrights**

Copyright for this article is retained by the author(s), with first publication rights granted to the journal.

This is an open-access article distributed under the terms and conditions of the Creative Commons Attribution license (http://creativecommons.org/licenses/by/4.0/).