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On Accumulation of the Budget Deficit: Spirit of MMT Through Mathematical Analysis

Yasuhito Tanaka

Faculty of Economics, Doshisha University, Kamigyo-ku, Kyoto, 602-8580, Japan

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Abstract

It is said that public finance must be balanced at least in the long run. According to the so-called MMT (Modern Money Theory or Modern Monetary Theory) approach, however, this is not true. It is often pointed out that MMT lacks the mathematical analysis used in ordinary economic discussions. The purpose of this paper is to present a brief theoretical and mathematical basis to the backbone of the MMT argument, while maintaining the basics of the neoclassical microeconomic framework, such as maximizing consumer utility through utility functions and budget constraints, and equilibrium between demand and supply of goods under perfect competition with constant returns to scale technology. Using a simple overlapping generations (OLG) model that includes economic growth due to technological progress, we present the following results. The budget deficit equals the increase in people's savings, and the accumulated budget deficit equals people's savings. The budget deficit is a cause and the savings is a consequence, not the other way around. Deficits are created by the government, which determines income, which determines savings. Deficits create savings, not savings finance deficits. Reducing the budget deficit will reduce savings, income, and consumption.

Keywords: Fallacy of fiscal discipline, MMT, economic growth, budget deficit

1. Introduction

The outstanding amount of government bonds in Japan is over 1200 trillion yen, which is said to be in a critical situation. It is said that public finance must be balanced at least in the long run. On the other hand, there is a view that there is no problem in accumulating budget deficits if the debt is not owed to foreign countries, and that fiscal policy should be evaluated only in terms of its effects, such as preventing inflation and achieving full employment and stable economic growth. The so-called Functional Finance Theory by Lerner (1943, 1944), is one such theory. MMT (Modern Money Theory or Modern Monetary

Theory, Wray (2015), Mitchell, Wray and Watts (2019) and Kelton(2020)), which has spread in the U.S. in recent years and has become a hot topic in Japan, is another representative example, and the proponents of MMT themselves admit that Lerner's theory is the source of their thinking[1]. However, it is often pointed out that MMT lacks theoretical analysis based on mathematical models compared to mainstream economics, which is based on a neoclassical framework.

In this paper, we examine the idea of the effects of budget deficit, which is the backbone of Functional Finance Theory and MMT's arguments, using a very simple mathematical model, while maintaining the basics of the neoclassical microeconomic framework, such as utility maximization of consumers by utility function and budget constraint, and equilibrium of supply and demand of goods. We consider production with constant returns to scale technology in a perfectly competitive industry.

Using an overlapping generations model with two periods under economic growth by technological progress, we will show

The budget deficit equals the increase in people's savings, and the accumulated amount of the budget deficit equals their savings. The budget deficit is the cause and the savings is the result, not the other way around. The budget deficit is created by the government, which in turn determines income, which in turn determines savings. The budget deficit creates the savings, not that the savings finance the budget deficit. Reducing the budget deficit will reduce savings, income, and consumption.

We consider the following several cases.

Case 1: Period 1 in which there is no older generation consumer.

Case 2: Period 2 in which there are older and younger generation consumers, and the younger generation consumers are fully employed. The economy grows under constant price.

Case 3: Period 3 in which there are older and younger generation consumers, and the younger generation consumers are fully employed. The excess budget deficit causes inflation.

Case 4: Period 3 in which there are older and younger generation consumers. However, a shortfall in the budget deficit creates a recession with involuntary unemployment.

Case 5: Period 4 after Period 3 of Case 4 in which a budget deficit which is larger than the level necessary and sufficient to maintain full employment under constant price without recession is used to overcome the recession and achieve full employment. We increase fiscal spending in this case.

Case 6: Period 4 after Period 3 of Case 4 in which we recover recession by tax reduction policy instead of an increase in fiscal spending.

The next section provides an overview of the model and an analysis of Case 1. In Sections 3-7, Cases 2-6 will be considered.

[1] Other references of MMT are Mochizuki (2020), Morinaga (2020), Nakano (2020) and Park (2020). These are introductory texts of MMT written in Japanese.

2. Case 1: Period 1 With No Older Generation Consumer

We use a two periods overlapping generations (OLG) model which is a simplified version of the model used by Otaki (2007, 2009, 2015). We assume that the good is produced under perfect competition although Otaki considered monopolistic competition. Consumers live over two periods, the younger period and the older period. They work only in the younger period. In the older period they consume the good by their savings carried over from their younger period. The ratio of consumption of the consumers in the younger period is α, and that in the older period is $1-\alpha$. We assume $\frac{1}{2} < \alpha < 1$. α is the marginal propensity to consume. These values are obtained by utility maximization of consumers by the Cobb-Douglas utility function about consumptions in the younger period and the older period.

There is one good produced under perfect competition with constant returns to scale technology. The labor productivity and the nominal wage rate in a period (Period 1) is 1. The economy grows by technological progress. The labor productivity increases at the rate $\gamma - 1 > 0$ from a period to the next period. The nominal wage rate also increases at the rate $\gamma - 1$ with constant price. Let L and L_f be the employment and the labor supply (or the employment in the full employment state). Let G and T be the government expenditure and the tax in Period 1.

In Period 1 there is no older generation consumer. The world starts from here. When the employment in Period 1 is L, the total supply of the good is L. If the younger generation consumers are fully employed, we have $L = L_f$. The savings of the younger generation consumers in Period 1 is

$$(1-\alpha)(L-T).$$

This is equal to the consumption of the older generation consumers in Period 2. The consumption of the younger generation consumers is

$$\alpha(L-T).$$

The total demand is

$$\alpha(L-T)+G.$$

From the equilibrium between the total supply and the total demand, we get

$$L = \alpha(L-T)+G.$$

Then,

$$G-T = (1-\alpha)(L-T). \qquad (1)$$

This implies

$$\text{budget deficit} = \text{savings of the younger generation consumers.}$$

In (1) the independent variables are G and T which are determined by the government. Therefore, the savings of the consumers is determined in dependence on the budget deficit. We get the following proposition.

Proposition 1 In Period 1, the world starts from here, in which there is no older generation consumer, the budget deficit is equal to the savings of the younger generation consumers, and the savings is determined in dependence on the budget deficit.

3. Case 2: Period 2 With Full Employment Under Constant Price

Suppose that in Period 1 the younger generation consumers were full employed, and also in Period 2 full employment is achieved. Let γG and γT be the government expenditure and the tax in Period 2. The price of the good and the nominal wage rate are 1. The total supply in Period 2 is

$$\gamma L_f.$$

Note that the growth rate is $\gamma - 1$. The savings of the younger generation consumers in Period 1 is

$$(1-\alpha)(L_f - T).$$

This is equal to the consumption of the older generation consumers in Period 2. The consumption of the younger generation consumers in Period 2 is

$$\alpha\gamma(L_f - T).$$

The total demand is

$$\alpha\gamma(L_f - T) + (1-\alpha)(L_f - T) + \gamma G.$$

From the equilibrium between the total supply and the total demand, we get

$$\gamma L_f = \alpha\gamma(L_f - T) + (1-\alpha)(L_f - T) + \gamma G. \tag{2}$$

Then,

$$\gamma(G-T) = \gamma(1-\alpha)(L_f - T) - (1-\alpha)(L_f - T) = (\gamma - 1)(1-\alpha)(L_f - T). \tag{3}$$

This implies

$$\text{budget deficit} = \text{savings of the younger generation consumers}$$

$$- \text{savings of the older generation consumers.}$$

If $\gamma = 1$, budget deficit $= 0$. This is because if the economy does not grow, the savings of the younger generation consumers are constant under full employment with constant price. In (3) the independent variables are G and T which are determined by the government. Therefore, the increase in the savings of the consumers is determined in dependence on the budget deficit.

If the savings earn interest at the rate *r*, and the government pays it, then (2) is

$$\gamma L_f = \alpha\gamma(L_f - T) + (1-\alpha)(1+r)(L_f - T) + \gamma G,$$

and we have

$$\gamma(G-T) + (1-\alpha)r\big(L_f - T\big) = \gamma(1-\alpha)\big(L_f - T\big) - (1-\alpha)\big(L_f - T\big)$$
$$= (\gamma - 1)(1-\alpha)(L_f - T).$$

This means

$$\text{budget deficit including payment of interest} = \text{savings of the younger generation consumers}$$

$$- \text{savings of the older generation consumers.}$$

We get the following proposition.

Proposition 2. In Period 2 with full employment under constant price the budget deficit is equal to the increase in the savings of the younger generation consumers, that is, the difference between the savings of the younger generation consumers in Period 2 and those in Period 1. The increase in the savings of the consumers is determined in dependence on the budget deficit.

4. Case 3: Period 3 with Full Employment and Inflation by Excess Budget Deficit

Suppose that in Period 2 full employment were achieved under constant price. Let p be the price and the nominal wage rate in Period 3. The government expenditure is ζG. We assume that the nominal value of the tax is $\gamma^2 T$. The nominal total supply in Period 3 is

$$p\gamma^2 L_f.$$

The nominal total demand is

$$\alpha\gamma^2(pL_f - T) + (1-\alpha)\gamma(L_f - T) + \zeta G.$$

From the equilibrium between them, we obtain

$$p\gamma^2 L_f = \alpha\gamma^2(pL_f - T) + (1-\alpha)\gamma(L_f - T) + \zeta G,$$

and then

$$\zeta G - \gamma^2 T = (1-\alpha)\gamma^2(pL_f - T) - (1-\alpha)\gamma(L_f - T). \quad (4)$$

Comparing this with (3),

$$(\zeta G - \gamma^2 T) - \gamma^2(G - T) = (1-\alpha)[\gamma^2(pL_f - T) - \gamma^2(L_f - T)].$$

This is rewritten as

$$(\zeta - \gamma^2)G = (1-\alpha)(p-1)\gamma^2 L_f.$$

When $(\zeta G - \gamma^2 T) - \gamma^2(G - T) = (\zeta - \gamma^2)G > 0$, we have $p > 1$.

The left hand side of (4) is the (nominal) budget deficit in this period, and its right hand side is the difference between the (nominal) savings of the younger generation consumers in Period 3 and the savings of the younger generation consumers in Period 2. In (4) the independent variables are G and T which are determined by the government. The increase in the savings of the consumers is determined in dependence on the budget deficit.

We obtain the following results.

Proposition 3 In an economy that grows at a constant rate through technological progress while maintaining full employment, inflation will occur if the budget deficit (given tax or given fiscal spending[2]) is larger than the level necessary and sufficient to sustain full employment under growth with

[2] A change in fiscal spending and that in tax have different effects on the national income. An increase in fiscal spending and that in tax while keeping the budget deficit constant will lead to an inflation,

constant price.

Even in this case the (nominal) budget deficit is equal to the increase in the savings of the younger generation consumers, that is, the difference between the savings of the younger generation consumers in Period 3 and the savings of the younger generation consumers in Period 2. The increase in the savings of the consumers is determined in dependence on the budget deficit.

5. Case 4: Recession and Involuntary Unemployment by Insufficient Budget Deficit

Suppose again that in Period 2 full employment were achieved under constant price. Let L be the employment in Period 3. Assume that the government expenditure is ζG, the tax is $\gamma^2 T$, and $p = 1$. The total supply in Period 2 is

$$\gamma^2 L.$$

The total demand is

$$\alpha\gamma^2(L - T) + (1 - \alpha)\gamma(L_f - T) + \zeta G.$$

From the equilibrium between them, we obtain

$$\gamma^2 L = \alpha\gamma^2(L - T) + (1 - \alpha)\gamma(L_f - T) + \zeta G,$$

and then

$$\zeta G - \gamma^2 T = (1 - \alpha)\gamma^2(L - T) - (1 - \alpha)\gamma(L_f - T). \quad (5)$$

Comparing this with (3),

$$(\zeta G - \gamma^2 T) - \gamma^2(G - T) = (1 - \alpha)[\gamma^2(L - T) - \gamma^2(L_f - T)] = (1 - \alpha)\gamma^2(L - L_f).$$

When $(\zeta G - \gamma^2 T) - \gamma^2(G - T) < 0$, $L < L_f$ is derived. Then, there is involuntary unemployment.

The left hand side of (5) is the budget deficit in this period, and its right hand side is the difference between the savings of the younger generation consumers in Period 3 and the savings of the younger generation consumers in Period 2. In (5) the independent variables are G and T which are determined by the government. The increase in the savings of the consumers is determined in dependence on the budget deficit.

We have proved the following proposition.

Proposition 4 If the actual budget deficit is smaller than the level necessary and sufficient to sustain full

while a decrease in fiscal spending and tax cut while keeping the budget deficit constant will lead to a recession. It is because the multiplier of a change in tax is smaller than that of a change in fiscal spending. However, a change in tax has an effect on consumption of the older generation consumers in the next period. We can show that an increase in fiscal spending and tax reduction have the same effects in the long run.

employment and growth at constant price (given tax or given fiscal spending[3]), there will be a recession and involuntary unemployment.

Even in this case the budget deficit is equal to the increase in the savings of the younger generation consumers, that is, the difference between the savings of the younger generation consumers in Period 3 and the savings of the younger generation consumers in Period 2. The increase in the savings of the consumers is determined in dependence on the budget deficit.

6. Case 5: Recovery from Recession by Fiscal Spending in Period 4

We will recover full employment in Period 4. Suppose that the taxes in Period 3 and 4 are $\gamma^2 T$ and $\gamma^3 T$. The total supply is

$$\gamma^3 L_f.$$

The consumption (or the savings) of the older generation consumers is

$$(1-\alpha)\gamma^2(L-T).$$

The consumption of the younger generation consumers is

$$\alpha\gamma^3(L_f-T).$$

Let ζG be the government expenditure. The total demand is

$$\alpha\gamma^3(L_f-T)+(1-\alpha)\gamma^2(L-T)+\zeta G.$$

Note that L is employment in Period 3.

From the equilibrium between the total supply and the total demand, we have

$$\gamma^3 L_f = \alpha\gamma^3(L_f-T)+(1-\alpha)\gamma^2(L-T)+\zeta G. \quad (6)$$

On the other hand, if there is no recession, in Period 4 (2) means

$$\gamma^3 L_f = \alpha\gamma^3(L_f-T)+(1-\alpha)\gamma^2(L_f-T)+\gamma^3 G. \quad (7)$$

By (6) and (7),

$$\zeta G-\gamma^3 G=(1-\alpha)\gamma^2(L_f-L). \quad (8)$$

When $L<L_f$, we have $\zeta>\gamma^2$. Thus, larger budget deficit is needed than would be the case without a recession. From (6)

$$\zeta G-\gamma^3 T=(1-\alpha)\gamma^3(L_f-T)-(1-\alpha)\gamma^2(L-T).$$

The left hand side of this equation is the budget deficit, and its right hand side is the increase in the

[3] Please see footnote 2.

savings of the younger generation consumers from Period 3 to Period 4. Similarly to the previous cases, the independent variables are G and T which are determined by the government. The increase in the savings of the consumers is determined in dependence on the budget deficit.

We obtain the following results.

Proposition 5 In order to restore full employment from a recession that includes involuntary unemployment caused by a shortfall in the budget deficit, a larger budget deficit is required than would be the case if full employment were maintained continuously.

Even in this case the budget deficit is equal to the increase in the savings of the younger generation consumers, that is, the difference between the savings of the younger generation consumers in Period 4 and the savings of the younger generation consumers in Period 3. The increase in the savings of the consumers is determined in dependence on the budget deficit.

7. Case 6: Recovery from Recession by the Tax Reduction in Period 4

Again we will recover full employment in Period 4. Suppose that the government expenditures in Period 3 and 4 are $\gamma^2 G$ and $\gamma^3 G$.

The total supply is

$$\gamma^3 L_f.$$

The consumption (or the savings) of the older generation consumers is

$$(1-\alpha)\gamma^2(L-T).$$

Let ηT be the tax in Period 4. Then, the consumption of the younger generation consumers is

$$\alpha(\gamma^3 L_f - \eta T).$$

The total demand is

$$\alpha(\gamma^3 L_f - \eta T) + (1-\alpha)\gamma^2(L-T) + \gamma^3 G.$$

Note that L is employment in Period 3.

From the equilibrium between the total supply and the total demand, we have

$$\gamma^3 L_f = \alpha(\gamma^3 L_f - \eta T) + (1-\alpha)\gamma^2(L-T) + \gamma^3 G. \quad (9)$$

Again if there is no recession, (7) holds in Period 4. By (7) and (9),

$$\alpha(\gamma^3 - \eta)T = (1-\alpha)\gamma^2(L_f - L). \quad (10)$$

When $L < L_f$, we get $\eta < \gamma^3$. Compare (8) and (10). Since $\alpha < 1$, we need larger value of tax reduction than an increase in fiscal spending in Case 5. This is because the propensity to consume is

smaller than 1. However, as noted in footnote 2, tax reduction affects the consumption in the next period, and we can show that in the long run tax reduction and an increase in fiscal spending have the same effects. From (9)

$$\gamma^3 G - \eta T = (1-\alpha)(\gamma^3 L_f - \eta T) - (1-\alpha)\gamma^2 (L - T)$$

The left hand side of this equation is the budget deficit, and its right hand side is the increase in the savings of the younger generation consumers from Period 3 to Period 4. Similarly to the previous cases, the independent variables are G and T which are determined by the government. The increase in the savings of the consumers is determined in dependence on the budget deficit.

We have shown the results almost the same as Proposition 5.

8. Concluding Remark

In this paper, we have examined MMT's claims about budget deficits using a simple overlapping generations model with production in a perfectly competitive industry and have found that they are generally correct.

By Proposition 1 the budget deficit equals the savings of the consumers in the first period. Propositions 2-5 mean that after the first period the budget deficit equals the increase in the savings of the younger generation consumers from a period to the next period. Therefore, the accumulated value of the budget deficit equals the savings of the consumers. Reducing the budget deficit will reduce savings, income, and consumption.

It is important to note that tax is not a source of revenue for fiscal spending, but that fiscal spending has a role to increase the demand for goods, on the other hand tax has a role to reduce the demand for goods by reducing people's income, and the budget deficit is merely the difference between the resulting fiscal spending and tax.

In this paper, we have used a simple two periods overlapping generations model. It is possible to generalize the analysis in this paper using the three periods or three generations overlapping generations model which includes a childhood period. Also, we can introduce pension system into the model.

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References

Kelton, S. (2020). *The Deficit Myth: Modern Monetary Theory and the Birth of the People's ZEconomy*. Public Affairs.

Lerner, A. P. (1943). Functional finance and the federal debt. *Social Research, 10*, 38-51.

Lerner, A. P. (1944). *The Economics of Control: Principles of Welfare Economics*. Macmillan.

Mitchell, W., Wray, L. R., & Watts, M. (2019). *Macroeconomics*. Red Globe Press.

Mochizuki, S. (2020). *A book understanding MMT* (in Japanese, *MMT ga yokuwakaru hon*), Shuwa System.

Morinaga, K. (2020). *MMT will save Japan* (in Japanese, *MMT ga nihon wo sukuu*), Takarajimasha.

Nakano, A. (2020). A book to understand the key points of MMT (in Japanese, MMT no pointo ga yokuwakaru hon), Shuwa System.

Otaki, M. (2015). *Keynsian Economics and Price Theory: Re-orientation of a Theory of Monetary Economy*. Springer. https://doi.org/10.1007/978-4-431-55345-8_2

Otaki, M. (2007). The dynamically extended Keynesian cross and the welfare-improving fiscal policy. *Economics Letters*, *96*, 23-29. https://doi.org/10.1016/j.econlet.2006.12.005

Otaki, M. (2009). A welfare economics foundation for the full-employment policy. *Economics Letters*, *102*, 1-3. https://doi.org/10.1016/j.econlet.2008.08.003

Park, S. (2020). *The fallacy of fiscal collapse* (in Japanese, *Zaisei hatanron no ayamari*), Seitosha.

Randall, W. L. (2015). *Modern Money Theory: A Primer on Macroeconomics for Sovereign Monetary Systems*. Palgrave Macmillan; 2nd ed..