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Extended Mean - Variance Portfolio Optimization Model: A Comparative Study Among Swarm Intelligence Algorithms

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Abstract

Portfolio optimization is one of the important issues in the effective management of investment. There is plenty of research in the literature addressing these issues. Markowitz's primary portfolio selection model is a more suitable method to solve the model for obtaining fairly optimum portfolios. But, the problem of portfolio optimization is multi-objective in nature that aims at simultaneously maximizing the expected return of the portfolio and minimizing portfolio risk. The computational complexity increases with an increase in the total number of available assets. Therefore heuristic methods are more suitable for portfolio optimization in compare to deterministic methods. This research compares three well-known swarm intelligence algorithms (e.g. Ant Colony Optimization (ACO), Particle Swarm Optimization (PSO) and Artificial Bee Colony (ABC)) for portfolio optimization. The Sharpe ratio was used as one of the important criteria for this comparison. PSO outperformed other algorithms in portfolio optimization experiments. The results were also showed that the portfolios which were made of monthly data had performed better than the yearly data.

Keywords: Portfolio optimization, ABC, ACO, PSO

1. Introduction

The selection of an appropriate portfolio of assets to invest in the major concern for fund management companies as well as individual investors. Individual investments are the main actions that any person can carry out in their lives and the purpose of asset management is the determination of these variables in a way that will minimize risk and maximize returns. The optimal asset selection usually occurs in operations between risk and return. The identification of the effective edge of the portfolios allows the investors to achieve the highest expected return from their investments based on the degree of the expected risk. The improved stock analysis methods, especially in markets with very high numbers of shares, lead to the new methods in addition to the previous methods in order to find answers to maximize profits in the financial markets. So far, many models are presented to solve the problem of optimal assets and make an efficient edge. Although theoretically these models are solved by using the mathematical programming methods, there are many problems in regard to the assets selection in practice. Due to the present problems in solving the non-linear assets programming model, the appropriate portfolio selection using the linear mathematical programming models is more suitable to solve the assets selection. This research presents a comparative study of swarm intelligence techniques to solve Extended Mean-Variance Portfolio Optimization Model. The best techniques found in this study will help investors to select the optimal portfolios for better return of their investments.

1.1 Portfolio Management

The conception of an optimal portfolio of assets is not a new concept. In 1900, this term was first coined by Louis Bacheliers in his doctoral thesis in Paris. Unfortunately, the concept was less common among financial managers. The portfolio optimization requires primary skills of actuarial mathematics, elementary concepts of share value (price). Markowitz (1951) and Sharpe (1963) was introduced the modern concept of portfolio optimization. They were awarded Nobel Prize in Economics in 1990. This theory seems to be of high importance to financial managers and practitioners.

In 1952, Harry Markowitz stated the investment theory called Modern Portfolio Theory (MPT) under the uncertainty based on risk and average income of shares. This method is based on the argument that the investors should not invest all their capital on one type of asset, rather they should invest in a series of stocks or assets due to the probable loss of capital or profit in one series or set of stock market shares. The collection of all shares are known as stock share or portfolio. Although some investors emphasize to buy shares of an industry, while others may focus on different industry stocks (Mehregan, 2004)

The major tasks of Investment Management are portfolio analysis and portfolio management. The analysis of the securities relies on the estimation of each investment benefits, while the portfolio management includes the investments components analysis and investments retention management. In the recent decade, the discussion of choosing the stock for investment (stock portfolio analysis) has moved to portfolio management (Strong, 2000). The investment process includes a series of activities in which they eventually lead to the purchase of the real properties or securities that can have a risk and return.

1.1.1 Return

Return is composed of two parts i.e dividends and profits & losses of capital. The most important component of the return is the interest in the form of the periodic cash flows and can be in the form of the interest or dividends. The capital gains or losses are the important components dedicated to the return of the common stock for on long term bonds. It is also consistent for the other securities' fixed income and caused by an increase (or decrease) of the asset price. The sum of these two components makes the total returns of the securities (P. Jones, 2012).

1.1.2 Risk

The risk is called hazard in Farsi that has the potential to cause injury or damage. The encyclopedia defines venture capital investment as the calculated potential loss of the investment. The subject of risk is a combination of hazard and return that can be examined by the different approaches. The different risk sources are defined and interpreted in different ways by different researchers. Harry Markowitz (1952) proposed the quantitative definitions on the Numerical Index of Risk for the first time. He defined risk as the standard deviation of a multi-period variable. The definition clarifies that the possible changes in a particular index, whether positive or negative induces risks. Therefore it is possible that these changes make us losses or beneficiary in investment. In the Webster dictionary, the term risk is defined as danger or hazard or exposure to loss or damage. Hence, the risk is the probability of an adverse event (Degiannakis, S., Floros, C., and Livada, A., 2008). Risk (probability), change (standard deviation), and outcome (result) are defined as the expected risk (Rockafellar, R.T. and Uryasev, S., 2000). The relationship between expected return and risk is positive in modern stock share portfolio theory, which means the higher expected return is related to the higher accepted risk.

1.2 Swarm Optimization Techniques

Optimization problems play a very important role in many scientific and engineering fields. In the last two decades, several swarm intelligence algorithms, such as ant colony optimization (ACO) (M. Dorigo and T. Stutzle, 2004; T. Liao, T. St ̈utzle, M.M.deOca, and M.Dorigo, 2014), particle swarm optimization (PSO) (J. Kennedy and R. C. Eberhart, 1995; D.Chen and C. Zhao, 2009), and artificial bee colony (ABC) algorithm (D.Karaboga, 2005; D. Karaboga and B. Basturk, 2008) have been developed for solving difficult optimization problem. Researchers have shown that algorithms based on swarm intelligent have great potential (M. H. Aghdam, N. Ghasem-Aghaee, and M. E. Basiri, 2009; B.Yagmahan and M.M. Yenisey, 2008; R. E. Perez and K. Behdinan, 2007) and have attracted much attention.

Ant Colony Optimization (ACO) is a meta-heuristic approach inspired by the Ant System (AS) proposed by Marco Dorigoin 1992 in his PhD thesis (Dorigo M., 1992; Dorigo M, Birattari M, Stutzle T., 2006; Pei Y, Wang W, Zhang S., 2012). It is inspired by the foraging behaviour of real ants. This algorithm consists of four main components (ant, pheromone, daemon action, and decentralized control) that contribute to the overall system. Ants are imaginary agents that are used in order to mimic the exploration and exploitation of the search space. In real life, the pheromone is a chemical material spread by ants over the path they travel and its intensity changes over time due to evaporation. In ACO the ants drop pheromones when travelling in the search space and the quantities of these pheromones indicate the intensity of the trail. The ants choose the direction based on the path marked by the high intensity of the trail. The intensity of the trail can be considered as a global memory of the system. Daemon actions are used together global information which cannot be done by a single ant and uses the information to determine whether it is necessary to add extra pheromone in order to help the convergence. The decentralized flexible within a dynamic environment.

Artificial Bee Colony (ABC) is proposed by D. Karaboga in 2005 for real parameter optimization (D.Karaboga, 2005). It is inspired by the intelligent behaviour of honey bees. Karaboga and Basturk have investigated the performance of the ABC algorithm on unconstrained numerical optimization problems (B. Basturk, and D. Karaboga, 2006; D. Karaboga, and B. Basturk, 2007; D. Karaboga, and B. Basturk, 2008) and its extended version for the constrained optimization problems (D. Karaboga, and B. Basturk, 2007). Hybrid artificial bee colony-based approach for optimization of multi-pass turning operations is being used in solving optimization (A.R. Yildiz, 2013).

Particle Swarm Optimization algorithm (PSO) is proposed by James Kennedy and Russell Eberhart in 1995 (Kennedy, J. and Eberhart, R., 1995). PSO is one of the most used EAs. It is motivated by the social behaviour of organisms such as bird flocking and fish schooling (Eberhart, R. C., and Kennedy, J., 1995). The PSO algorithm, while making an adjustment towards "local" and "global" best particles, is similar to the crossover operation used by genetic algorithms (Eberhart, R. C. and Shi, Y., 1998).

Today, a wide range of stocks selection methods and models, e.g. Technical, Fundamental and Modern Portfolio Theory are available for investors (Eslami Bidgoli, 1995). The stock market behaviours are nonlinear similar to many natural phenomena. The linear models are incapable of correct diagnosis of nonlinear and linear parts and can only recognize the good behaviour linearity. Thus they need the non-linear models to predict the future behaviour of the effective equity stake and make appropriate decisions.

2. Portfolio Optimization Problem

The standard Markowitz mean-variance portfolio selection problem is formulated as follows (Markowitz, 1959):

$$\text{Min } \sigma_{R_p}^2 = \sigma_p^2 = \sum_{i=1}^{N} \sum_{j=1}^{N} W_i\, W_j\, Cov(R'_i, R'_j) \tag{1}$$

Subject to:

$$R'_p = E\left(R_p\right) = \sum_{i=1}^{N} w_i\, R'_j \geq R \tag{2}$$

$$\sum_{i=1}^{N} w_i = 1 \tag{3}$$

$$w_i \geq 0 \;\; \forall i \in \{1,2,\dots,N\} \tag{4}$$

Where N is the number of differently available assets, R'_i is the mean return of asset i; $Cov(R'_i, R'_j)$ is the covariance of returns of assets i and j, and R is the investor’s expected rate of return? The decision variable w_i represents the proportion of capital to be invested in asset 'i'. The total available budget invested is ensured by eqn (3). The goal is to minimize the portfolio risk (σ_p^2), for a given value of portfolio expected return (R'_p). Bounds on holdings and cardinality constraints are required for optimizing the portfolios (Chang et al., 2000;

Kellerer & Maringer, 2001). The former guarantee that the amount invested (if any) in each asset is between its predetermined upper and lower bounds while the latter ensures that the total number of selected assets in the portfolio is equal to a predefined number. Two other important constraints, namely, minimum transaction lots and sector capitalization were added to make the model more accurate (Chang et al., 2000; Soleimani, 2007; Soleimani, Golmakani, & Salimi, 2009). The minimum transaction lots constraint requires that each asset can only be purchased in batch with a given number of units (Chang et al., 2000); Oh et al., 2006; Soleimani et al., 2009). Investors tend to invest in assets belong to the sectors with higher a value of market capitalization to reduce their risk of investman ent (Soleimani et al., 2009). The extended mean–variance model for the portfolio selection problem is, thus, formulated as follows:

$$\text{Min } \sigma^2_{R_p} = \sum_{i=1}^{N} \sum_{j=1}^{N} W_i W_j \, Cov(R'_i, R'_j) \tag{5}$$

Where

$$W_i = \frac{x_i c_i z_i}{\sum_{j=1}^{N} x_i c_i z_i}, \ \text{i} = 1, \dots, \text{N} \tag{6}$$

Where

$$\sum_{i=1}^{N} Z_i = \text{M} \le N; M, N \in \mathbb{N}, \forall i = 1, \dots, N \ z_i \epsilon \{0,1\} \tag{7}$$

Subject to

$$\sum_{i=1}^{N} x_i c_i z_i R'_i \ge BR \tag{8}$$

$$\sum_{i=1}^{N} x_i c_i z_i \le B \tag{9}$$

$$0 \le B_{LOW_i} \le x_i c_i \le B_{UP_i} \le\le B, \quad i = 1, \dots, N \tag{10}$$

$$\sum_{i_s} W_{i_s} \ge \sum_{i_{s'}} W_{i_{s'}} \tag{11}$$

$$\forall y_s, \forall y_{S'} \ne 0; S, S' \in \{1, \dots, S\}, s < s' \tag{12}$$

Where

$$y_s = \begin{cases} 1 & if \ \sum_{i_s} Z_i > 0 \\ 0 & if \ \sum_{i_s} Z_i = 0 \end{cases} \tag{13}$$

$$i_s i_{s'} \in \{1, \dots, N\} \tag{14}$$

Where

M	Number of assets to be selected out of available assets(N)

B	Total budget
B_{LOW_i}	The lower limit of budget invested in an asset 'i'
B_{UP_i}	The upper limit of budget invested in an asset 'i'
S	Total number of sectors
c_i	The minimum transaction lot for asset 'i'. (integer)
x_i	Number of c_i's that is purchased (integer)
$c_i\ x_i$	The number of units of asset 'i' in the selected portfolio
$z_{i=}$	$\begin{cases} 1, \text{if asset i is in the portfolio} \\ 0, \quad Otherwise \end{cases}$
$y_s =$	$\begin{cases} 1, \text{if the sector s has at least one selected asset} \\ 0, \quad Otherwise \end{cases}$
i_s	Set of asset indices which belong to Sectors'

With the above notations, the cardinality constraint is represented by inequality (7) and (8) is the same as (2). The Eq (9) is represented the budget constraint and easy of search, it was converted to inequality. Eq (10) is represented the bounds on holdings constraints and the sector capitalization constraints are induced by inequalities (11) and (12). The sector with more capitalization should have more proportion in the ultimate portfolio. This constraint is preferred when investing in assets belong to the sector with higher capitalization value. The assets with low return and/or high risks are excluded. Thus, to exclude such assets a 0/1 variable(y_s), is introduced and shown in Eq. (12) In addition to the above constraints, the sectors are sorted in descending order by their capitalization value. The extended model is a quadratic mixed-integer programming model and uses heuristics to find an efficient solution. In the next section, first PSO as a heuristics is reviewed and, then an efficient approach to solve the extended model using PSO is presented.

On the other hand, past researches generally focus on efficient frontier in portfolio optimization, but, this paper was attempted to optimize the portfolio Sharpe ratio (RVAR) (Sharpe, W. F., 1966). In RVAR, the mean and variance of an asset were combined in the Sharpe ratio. The ordinal scale was used to measure RVAR and the portfolios were gradable and comparable with this measure. The following equation was used to measure RVAR:

$$RVAR = \frac{TR_{p(av)} - RF_{p(av)}}{SD_p} \tag{15}$$

Where

$TR_{p(av)}$	Mean return of the portfolio
$RF_{p(av)}$	The rate of return of risk-free security (16% was considered for this research)
SD_p	Standard deviation of $TR_{p(av)}$

The main objective was to maximize the portfolio RVAR (modifying the portfolio's weight (w_i)) by balancing two contradicting objectives i.e maximizing the expected return and minimizing the risk. RVAR was calculated in this research by applying PSO, ACO, and ABC to identify the best combination of selected shares/stocks in the given portfolio.

3. Swarm Intelligence Algorithms

3.1 Artificial Bee Colony (ABC)

Artificial Bee Colony (ABC) algorithm was proposed by Karaboga (2005) for real parameter optimization (D. Karaboga, 2005). It is inspired by the intelligent behaviour of honey bees. The colony of artificial bees consists of three groups of bees: employed, onlooker and scout bees. Half of the colony composed of employed bees and the rest consist of the onlooker bees. The number of food sources/nectar sources is equal with the employed bees, which means one nectar source is responsible for one employed bee. The aim of the whole colony is to maximize the nectar amount. The duty of employed bees is to search for food sources (solutions). Later, the nectars "amount (solutions" qualities/fitness value) is calculated. Then, the information obtained is shared with the onlooker bees which are waiting in the hive. The onlooker bees decide to exploit a nectar source depending on the information shared by the employed bees. The onlooker bees also determine the source to be abandoned and allocate its employed bee as scout bees. For the scout bees, their task is to find the new valuable food sources. They search the space near the hive randomly (D. Karaboga, 2005). In the ABC algorithm, suppose the solution space of the problem is D-dimensional, where D is the number of parameters to be optimized. The fitness value of the randomly chosen site is formulated as follows

$$fit_i = {}^{1}/_{(1 + F_i)} \tag{16}$$

Where, F_i is the objective function. 'SN' represents the size of employed bees and onlooker bees and is equal to the number of food sources. For each food source, there is only one employed bee. The first position of the employed bee is randomly generated. After that, in each iteration, the employed bee determines a new neighboring food source and computes the nectar amount using the following equation:

$$v_{ij} = x_{ij} + \emptyset(x_{ij} - x_{kj}) \tag{17}$$

Where

$$i = 1,2, \dots, SN, j = 1,2,, D \ \& \ \emptyset = random\ number\ in\ range\ [0,1]$$

The bee occupied the new position if the new food source is better than that of the previous one. All the employed bees shared their new position to onlooker bees after each iteration. Then the onlooker bee evaluates the nectar information and chooses a food source usingP_i:

$$P_i = {fit_i}/{\sum_{j=1}^{SN} fit_j} \tag{18}$$

Where fit_i is the fitness value of the solution i. Later, the onlooker bee searches a new solution in the selected food source site, the same way as exploited by employed bees. After all the employed bees exploit a new solution and the onlooker bees are allocated a food source, if a source is found that the fitness hasn't been improved for a predetermined number of cycles (limit parameter), it is abandoned, and the employed bee associated with that source becomes a scout bee. In that position, scout generates randomly a new solution by:

$$x_i^j = x_{min}^j + r(x_{max}^j - x_{min}^j) \tag{19}$$

Where;

r is a random number in the range [0, 1].

$x_{max}^j\ and\ x_{min}^j$, are the lower and upper borders in the jth dimension of the problem space.

3.2 Particle Swarm Optimization Algorithm (PSO)

PSO is a heuristic search method which is derived from the behaviour of social groups like bird flocks or fish swarms **(**D. N. Wilke, 2005)**.** PSO moves from a set of points to another set of points using a combination of deterministic and probabilistic rules in a single iteration. The PSO has been popular in optimization due to its ease of implementation. It has the ability to effectively solve highly nonlinear, mixed integer optimization problems. Optimization is achieved by giving each individual in the search space a memory to store its previous information about the successes of a social group. The knowledge gain from the previously stored information helps the movement of the individual in the group (D. N. Wilke, 2005). Therefore, each individual (called particle) is characterized by its position $\underset{x\ i}{\rightarrow}$, its velocity $\underset{v\ i}{\rightarrow}$, its personal best position and its neighborhood best position $\underset{p\ i}{\rightarrow}$ The elements of the velocity vector for particle 'i' are updated as

$$v_{ij} = \omega v_{ij} + c_1 q\left(x_{ij}^{pb} - x_{ij}\right) + c_2 \gamma\left(x_j^{sb} - x_{ij}\right) \tag{20}$$

Where,

w	Inertia weight
x_{ij}^{pb}	Best variable vector encountered so far by particle
x_j^{sb}	Swarm best vector
γ	Random numbers in the range (0, 1)

The variable vector of the particle 'i' is modified after the velocity update according to the following equation:

$$x_{ij} = x_{ij} + v_{ij} \tag{21}$$

The above cycle of evaluation followed by the updates of velocities and positions (x_{ij}^{pb} and x_j^{sb}) is then repeated until a satisfactory solution has been achieved.

3.3 Ant Colony Optimization (ACO) Algorithm

Ant Colony Optimization (ACO) algorithm is a new kind of simulated evolutionary algorithm has been successfully applied to several NP-hard combinatorial optimization problems (Huang L, Chen H, Hu T, 2013; Tang R, Qin Y, Zhang L, 2011). It is based on the food-seeking behaviour of ants in 1996. The ants release some pheromone on their way and they never walked when these ants reach a crossing. From the crossing, they choose a path randomly and release some pheromone proportional to the length of the path. The following ants follow the trail of the other ants to the food source by sensing the pheromone on the path. As this process continues, most of the ant likely to choose the shortest path with a huge amount of pheromones. During this occasion a mechanism of positive feedback is formed, this mechanism ensures that the good information can be preserved and finally ants can find an optimal (Liang Bai, Y.L. Hu, S.Y, Lao, W.M, Zhang, 2010; B. Tang, Y.Y, Yin, Quan, Liu, Z.D. Zhou, 2008). In this case, there are more and more pheromones on this path. As the time on, the amount of information on other paths will be gradually reduced, eventually, the path most ants moved to will be the optimal path.

Let apply the ACO algorithm to find the shortest distance between two cities among ‘n’ cities. Suppose there are’ numbers of ants. Firstly the algorithm set any ants into some randomly selected cities. Ant *k (k=1, 2, 3,…, m)* will determine the transfer direction according to the concentration of the pheromone on each path in the searching for target city. At first, the ants will randomly select a path because of the minor difference of the pheromone quantity among paths. The tabu list $tabu_k$ ($k=1, 2, 3,..., m$) is to record the path which ant k has walked and it will adjust dynamically along with the changing movement of the ant.

Let $P_{ij}^k(t)$ be the state of transition probability of ant 'k' for selecting city 'j' as the target city at a moment 't'.

$$P_{ij}^k(t)=\begin{cases}\frac{[\tau_{ij}(t)]^{\propto}[\partial_{ij}(t)]^{\beta}}{\sum_{s\,c\,allowed_k}[\tau_{is}(t)]^{\propto}[\partial_{is}(t)]^{\beta}}, & if\ j\ \epsilon\ allowed_k \\ 0, & else\end{cases} \quad (22)$$

In the above equation, $allowed_k$ represents the allowed cities of ant k in the next step. $\tau_{ij}(t)$ represents the pheromone of the path between city i and city j at time t. At the initial time, the pheromone of each path is equal. $\partial_{ij}(t)$ is a heuristic function and defined as

$$\partial_{ij}(t) = {}^{1}/_{d_{ij}} \quad (23)$$

Where d_{ij} (the distance between nodes) to represent the desired transfer degree of ants from the city i to the city j. α is the heuristic factor of pheromone, which indicates the relative importance of the track. It reflects the guiding effect of the information accumulated in the track on the ant's movement. The greater the values is, the more likely the ant chooses the track which other ants passed by. β is expected heuristic factor, which indicates the relative weight of calculation ability. It reflects the importance of degthe ree of heuristic information in ant's choice. The greater the value is, the closer the state transition probability to the greedy rule.

4. Experimental Results

This study compares three Swarm Intelligence algorithms e.g. PSO, ABC and ACO algorithms to select optimum portfolio. Returns of top 50 stocks from BSE were selected as a case study. All the data was collected from March 2015 to November 2018. The date from March 2015 to March 2018 was used for selecting portfolio and trains the model and data from April 2018 to September 2018 was used for evaluating the performance of portfolios using the trained model. The comparisons of the results are performed based on the following four criteria.

(1) Expected Return of Portfolio,

(2) The variance of the portfolio,

(3) Mean returns of the portfolio,

(4) Std. Dev. of the portfolio's returns,

(5) Sharpe ratio

4.1 Experimental Parameters

Codes of all the algorithms are implemented using python. Before running the algorithms, its parameters should be determined based on literature and past research. These values have been determined in this study as specified below:

Table 1. PSO parameters

Parameters	**Values**
Number of particles	50
Maximum number of cycles (MCN)	100
The individual and social learning rate	2
r_1, r_2 (Random numbers using uniform distribution)	[0,1]

Table 2. ABC parameters

Parameters	**Values**
Maximum number of iterations	100
Colony size	50
Number of spectators and the worker bees	50% of Colony Size
Number of scout bees	1
λ	0.5

Table 3. ACO parameters

Parameters	**Values**
Number of Ants	168
Q	0.1
Evaporation rate (γ)	0.01
Repeat count	500
Coefficient vector (c)	[0,1,0,2,0,3,0,4]

After being assured of the stability of all the algorithms, the mean-variance model was executed on monthly and yearly input data. In order to evaluate the performance of the portfolio, the six-month return of top '50' companies was used as test data. The return of portfolios on the six-month test period and breakdown of each month were shown in Table 4. For the month of April 'ABC,' based algorithm showed more promising results than others.

PSO based algorithm performed better for the month May, June, July and September in comparison to other algorithms. Whereas ACO showed better results in August for both monthly and yearly input mode. From the above discussion, it was clear that in most of the cases PSO performed better than other algorithms.

Table 4. Performance of PSO, ACO, ABC on portfolio return

Return						**Input Mode**	**Algorithm**
April	May	June	July	August	September		
7.21	2.5	-3.09	0.24	4.56	10.15	Monthly	PSO
7.89	4.13	-3.67	-0.15	-0.09	15.17	Yearly	PSO
5.34	-0.13	-5.54	-1.78	6.17	12.78	Monthly	ACO
6.24	0.78	-5.78	-2.89	5.18	8.90	Yearly	ACO
12.6	1.78	-6.48	-2.56	3.67	9.16	Monthly	ABC
11.15	-0.89	-5.12	-0.56	3.89	10.16	Yearly	ABC

The results obtained in Table 4 and Eq. (3) was used to calculate RVAR. The expected return, the variance of portfolios, mean returns of the portfolio in the six-month test period, the standard deviation of returns in the test period and RVAR were shown in Table 5.

Table 5. Performance of PSO, ACO, ABC on different parameters

Input	Std. Dev. of the portfolio's returns	Mean returns of the portfolio	Variance of portfolio	Expected Return of Portfolio	RVAR	Algorithm
Monthly	9.76	5.67	0.001	0.889	0.51	PSO
Yearly	8.35	5.10	0.012	0.678	0.32	PSO
Monthly	7.90	4.89	0.001	0.010	0.31	ACO
Yearly	7.14	4.33	0.671	0.678	0.29	ACO
Monthly	9.10	5.19	0.001	0.071	0.31	ABC
Yearly	6.65	4.48	0.677	0.771	0.30	ABC

Results from Table 5 indicated that PSO outperformed ABC and ACO algorithm for RVAR. PSO also performed better for the return of the portfolio and expected return of portfolios.

5. Conclusion

In this paper, three swarm intelligence based algorithms e.g PSO, ABC, ACO were used to optimize the portfolio. All the algorithms were tested on monthly and yearly data of the top 50 listed companies in BSE. The Sharpe Ratio was mainly used for comparing different algorithm. The experimental results showed that the efficiency of the PSO in the portfolio optimization problem outperformed other algorithms. Again, monthly based portfolios were shown better performance than yearly data.

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