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One Iterative Method for calculating the Thermal Conductivity of Layered soil

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Abstract

The equation of heat conduction in a heterogeneous environment is being studied. An approximate method was developed for calculating the thermal conductivity of layered soil. Priori estimates were obtained to solve the direct and conjugate problems of difference. We prove the limitation of approximate values of the thermal conductivity coefficient. Numerical calculations were aimed to test the convergence of iterative calculative scheme of the thermal conductivity of the soil.

Keywords: dispersion environment, equation of heat conductivity, inverse problem, dual problem, priori estimates, thermo physical characteristics of the soil.

1. Overview

There is various ways of determining the thermal conductivity coefficient.

The principle of generalized conductivity proposed by Maxwell C. was not effective enough, because it was based on an idealized model structures. Such kind of approach ignores the real characteristics of disperse structures, namely: the structural and mechanical properties, size of particle and pores, the nature of contact between the particles, their degree of packing and total porosity of the system. Certainly, listed set of structural and mechanical properties determine the pattern of heat transfer in a dispersed system.

Path of analysis of the **structure of real dispersed materials** has been very fruitful. Along this route came Zichtenecker K., Krischer O., Ribaud M., Schuman T.E., Hängst C., Austin I.B., Smith W.O., Nerpin S.V. and Chudnovsky, A.D.

All these works lead to the formulas for the thermal conductivity of intermediate medium and disperse systems, expressing the latter in the form of the total porosity function of pores or particles. However, almost without exception, the calculated methods yield significant differences with experiment.

Significant values for estimating the thermal conductivity of the dispersed system is its structure. Therefore, all formulas can be reduced to two types: in one - dispersed body is treated as a granular material, in the other - like cellular.

In the first case, the body is solid particles - grains, which are separated by an intermediate environment (moisture, steam, air). To such structure called the statistical mixture, the most suitable formula is by Loeb I., Kaufman, B.N., Franchuk A.U. and A. Lilikova.

In the second case, related materials are composed of masses, which include the closed pores of different structures. For such structure, which is called matrix or joint, formula by Russel H.W. and Eucken A. is the most appropriate.

The most important factor determining the thermal properties of dispersed systems is a solid phase system, its skeleton. Experiments show that if the magnitude of the specific heat capacity of the various terms of the skeleton differs only in the range of 25%, the thermal conductivity of these differences reaches 1000% .

With the advent of fast computing machines approximate methods are rapidly developing to calculate the thermal conductivity of the soil. In this case, based on a mathematical model of heat distribution in the soil, the inverse problem of heat conduction equations is composed.

Hdamar J. gave an example of incorrect problem and formulated the concept of well-posed problems for differential equations. The founders of the theory of incorrect problems are Tikhonov, Ivanov V.K. and Lavrentiev M.M.

Classification of inverse problems for parabolic equations and their solution methods is described in the papers of Alifanov O., Beck J.V., Blackwell B., St. Clear Ch., Kabanikhin S.I., Bektemesov M.A., and Nechaev D.V.

Some difference methods for solving inverse problems of the heat equation were studied by Rysbaiuly B., Akishev T.B., Baymankulov A.T., Ismailov, A., and G. Makhambetova. For the first time the stability of difference schemes was studied and the theoretical basis of difference schemes is outlined in the work of A.A. Samarskii.

Differential equations of heat conduction. To solve critical issues about heat transfer through the body, we must find functions, which are particular solution of the basic differential equation of heat conduction:

$$div(\lambda\, grad\,\theta) = C\frac{\partial\theta}{\partial t}$$

Here, λ – thermal conductivity of the body, C – volumetric heat capacity of the body. Derivation of this equation is contained in the basic courses of the thermal conductivity.

Boundary conditions. The nature of the boundary conditions can be different, depending on the conditions in which the actual heat processes flow. In our case, we consider the problem of heat propagation in a multilayer region. In this case, the external environment is air.
Whatever environment was at its physical state or dynamic nature, to solve the problem of heat conduction it is important to know:
A) The variation of temperature with time at the boundaries of the body (1-type boundary conditions);
B) The balance of the heat or heat flux as a function of time at the border (2- type boundary conditions).

In the presence of gaseous and liquid environment around a solid body, the central role is played by convective heat transfer between the body and the environment. The heat balance equation is usually written in this case as a boundary condition of the 3-type:

$$q(t) = \alpha(\theta_G(t) - \theta_c(t)) = -\left.\frac{\partial\theta(t)}{\partial n}\right|_G .$$

Where α - the heat transfer coefficient, q-heat flux on the boundary surface (unit area per unit time), n - normal.
Initial conditions: In general, condition of the temperature distribution within the body in the directions x, y, z for the initial time t = 0 can be written as:

$$\theta(x, y, z, 0) = \theta_0(x, y, z).$$

Solution of equations of heat conduction in a disperse medium is very difficult, since it contains a large number of unknown parameters, which are difficult to take into account. The heat equation is derived by considering the actual specifics of a complex object, namely the nature of the effective thermal properties of the material. This means that we apply the same heat equation to non-metallic materials as in solid bodies, but with the complications introduced by these circumstances.

2. Statement of the Problem

In this paper we study one-dimensional problem of heat propagation in the soil. In general, any problem of heat propagation is three-dimensional, but if the width and length of the region are large enough, and the surface of this region is almost flat, then the gradient of the horizontal distribution of heat is almost zero. In this case, instead of three-dimensional problem can be studied one-dimensional problem.

Let in the area $Q=(0,H)\times(0,T)$ occurs heat distribution under the influence of the ambient temperature, in this case - the air. Numerous experiments demonstrated that the heat distribution in the soil can be described by the equation of heat conduction

$$C\frac{\partial\theta}{\partial t}=\frac{\partial}{\partial z}\left(\lambda(z)\frac{\partial\theta}{\partial z}\right) \quad . \tag{1}$$

At the boundary surface to air, the law of conservation of energy is valid

$$\lambda\left.\frac{\partial\theta}{\partial z}\right|_{z=H}+\alpha\left(\left.\theta\right|_{z=H}-T_b\right)=0 \tag{2}$$

It was found that at a certain depth of the earth, earth's temperature remains constant. Using this fact put a boundary condition

$$\theta(0,t)=T_1=c\,o\,n\,s \tag{3}$$

Note that the Oz axis is directed vertically upwards. At the initial moment of time, t = 0 the temperature distribution in the ground is given, i.e.

$$\theta(z,0)=\theta_0(z), \qquad 0\le z\le H \tag{4}$$

Consider the case where soil in the depth is composed of several layers. Without loss of generality we can assume that the soil is composed of three layers. In the transition from one layer to another, temperature and the temperature flow is continuous:

$$[\theta(z,t)]_{h_k}=0, \qquad \left[\lambda\frac{\partial\theta}{\partial z}\right]_{h_k}=0\,, \qquad \text{k=1, 2.} \tag{5}$$

Where h_k - the boundary coordinate of transition from one layer to another layer. The formula $[f]_{h_k}=f(h_k+0,t)-f(h_k-0,t)$ expresses discontinuity of the function at the point $z=h_k$ .In order to determine the thermal conductivity of the soil; moreover temperature on the earth surface is given

$$\theta_G=\theta_g(t), \quad 0<t<T \quad . \tag{6}$$

3. Approximate method

Write down the exact solution of (1) - (5) in the final form is not possible, so the task is solved approximately. For this, segment (0, N) is divided into N equal parts with a step $\Delta z = H/N$, and the interval (0, T) is divided into m equal parts with a step $\Delta t = T/m$. In the resulting grid area

$$Q_N^m = \{z_i = i\Delta z;\ i = 0,1,...,N;\quad t_j = j\Delta t;\quad j = 0,1,...,m\}$$

the difference problem is studied.

$$C(z_{i-0.5})Y_{i\bar{t}}^{J+1} = (\lambda(z_{i+0.5})Y_{ix}^{J+1})_{\bar{z}},\quad \text{i=1, 2,…, N-1;}\quad \text{j=0,1,…,m-1} \tag{7}$$

$$Y_0^{J+1} = 0,\ \ \lambda_{N-0.5}Y_{N\bar{z}}^{J+1} + \alpha(Y_N^{J+1} - T_b(t_{j+1})) = 0\quad \text{j=0, 1,…, m-1} \tag{8}$$

$$Y_i^0 = \theta_0(z_i)\quad,\ z_i = i*dz;\quad \text{i=0,1,…,N} \tag{9}$$

Here $z_{i-0.5} = (i-0.5)dz,\ \mathrm{i} = 1,2,...,\mathrm{N}$. We believe that the points of discontinuity z_k fall to a whole node. In this case condition (5) is automatically taken into account by the system (7). Above problem will be solved approximately. To do this, choose the initial approximation $\lambda_0(z)$. The next approximation from the condition

$$J(\lambda_{n+1}) < J(\lambda_n),\ \mathrm{n} = 0,1,2,...$$

Where

$$J(\lambda) = \sum_{J=0}^{m-1}\left|Y_N^{J+1} - \theta_g(t_{J+1})\right|^2 \Delta t\,. \tag{10}$$

This means that the solution of (7) - (9) depends on the approximate value of the thermal conductivity coefficient, that is $Y_i^{j+1} = Y_i^{j+1}(\lambda_n)$.

3.1. The dual problem

The system (7) holds for $\lambda_n(z)$ and $\lambda_{n+1}(z)$. We write this system for the difference $\Delta Y = Y_i^{j+1}(\lambda_{n+1}) - Y_i^{j+1}(\lambda_n)$:

$$C(z)\Delta Y_{i,\bar{t}}^{j+1} = \left(\Delta\lambda Y_{iz}^{j+1}(\lambda_{n+1}) + \lambda_n \Delta Y_{iz}^{j+1}\right)_{\bar{z}} \tag{11}$$

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$$\Delta\lambda Y_{N\bar{z}}{}^{j+1}(\lambda_{n+1})+\lambda_n\Delta Y_{N\bar{z}}^{j+1}+\alpha\Delta Y_N^{j+1}=0\,,\,\Delta Y_0{}^{j+1}=0\,,\quad \Delta Y_i{}^{0}=0 \qquad (12)$$

Multiplying (11) to $U_i^j\Delta z\Delta t$ and sum over all internal nodes of the grid Q_N^m and applying summation by parts and using condition (12) we obtain

$$\sum_{i=1}^{N-1}C(z)\Delta Y_i^m U_i^m\Delta z-\sum_{i=1}^{N-1}\sum_{j=0}^{m-1}\Delta Y_i^{j+1}C(z)U_{it}^{j+1}\Delta t\Delta z=-\sum_{j=0}^{M-1}\sum_{i=1}^{N}\left(\Delta\lambda Y_{i\bar{z}}{}^{j+1}(\lambda_{n+1})+\lambda_n\Delta Y_{i\bar{z}}^{j+1}\right)U_{i\bar{z}}^j\Delta z\Delta t-$$

$$-\sum_{j=0}^{m-1}\alpha\Delta Y_N^{j+1}U_N^j\Delta t-\sum_{j=0}^{m-1}\left(\Delta\lambda Y_{0z}{}^{j+1}(\lambda_{n+1})+\lambda_n\Delta Y_{0z}^{j+1}\right)U_0^j\Delta t$$

Assuming $U_i^m=0,\ \mathrm{U}_0^{\mathrm{j}}=0$ and again applying the formula of summation by parts we obtain

$$-\sum_{i=1}^{N-1}\sum_{j=0}^{m-1}\Delta Y_i^{j+1}\left(C(z)U_{it}^{j+1}+\left(\lambda U_{iz}^j\right)_{\bar{z}}\right)\Delta t\Delta z=-\sum_{j=0}^{M-1}\sum_{i=1}^{N}\Delta\lambda Y_{i\bar{z}}{}^{j+1}(\lambda_{n+1})U_{i\bar{z}}^j\Delta z\Delta t-$$

$$-\sum_{j=0}^{m-1}\alpha\Delta Y_N^{j+1}U_N^j\Delta t-\sum_{j=0}^{m-1}\Delta Y_N^{j+1}\lambda(z)U_{N\bar{z}}^j\Delta t \qquad (13)$$

Grid function U_i^j is chosen in a way that there is equality

$$C(z)U_{\bar{it}}^{j+1}+\left(\lambda U_{iz}^j\right)_{\bar{z}}=0,\mathrm{i}=1,2,...,\mathrm{N}-1;\ \mathrm{j}=0,1,2,...,\mathrm{m}-1$$

In addition, we set boundary condition

$$\lambda_n(z)U_{n\bar{z}}^{j+1}+\alpha U_N^j=2\left(Y_N^{J=1}(\lambda_n)-\theta_g^{j+1}\right)$$

Then from (13) follows important relation for further research

$$2\sum_{j=0}^{m-1}\Delta Y_N^{j+1}\left(Y_N^{j+1}(\lambda_n)-\theta_g^{j+1}\right)\Delta t=-\sum_{j=0}^{m-1}\sum_{i=1}^{N}\Delta\lambda Y_{i\bar{z}}^{j+1}(\lambda_n)U_{i\bar{z}}^j\Delta z\Delta t-\sum_{j=0}^{m-1}\sum_{i=1}^{N}\Delta\lambda\Delta Y_{i\bar{z}}^{j+1}U_{i\bar{z}}^j\Delta z\Delta t \qquad (14)$$

During the derivation of formula (14) the conjugate problem was obtained

$$C(z)U_{it}^{j+1}+\left(\lambda U_{iz}^j\right)_{\bar{z}}=0,\mathrm{i}=1,2,...,\mathrm{N}-1;\ \mathrm{j}=0\,,1\,,2\,,\mathrm{m}-1 \qquad (15)$$

$$U_i{}^m=0,\mathrm{i}=0,1,2,...,\mathrm{N};\quad ,\ U_0^j=0,\ \mathrm{j}=\mathrm{m}-1,\mathrm{m}-2,...,0; \qquad (16)$$

$$\lambda_n U^j_{N,\bar{z}} + \alpha U^j_N = 2\left(Y_N^{\;j+1} - \theta_g^{\;j+1}\right),\ \mathrm{j} = \mathrm{m}-1, \mathrm{m}-2, \ldots \tag{17}$$

3.2. Auxiliary relations

Method of a priori estimates proves the following lemma.

Lemma 1. If $\theta_0(z) \in L_2(0,H)$, to solve problems (7) - (9) we have estimation

$$\max_j \left\|\sqrt{C}Y^{j+1}\right\|^2 + \sum_j \left\|\sqrt{\lambda}Y^{j+1}_{i\bar{z}}\right\|^2 \Delta t \le \left\|\sqrt{C}\theta_0\right\|^2,$$

Here $\left\|Y^{j+1}\right\|^2 = \sum_{i=1}^{N-1}\left(Y_i^{j+1}\right)^2 \Delta z$, $\left\|\sqrt{\lambda}Y_{\bar{z}}^{j+1}\right\| = \sum_{i=1}^{N}\lambda\left(Y_{i\bar{z}}^{j+1}\right)^2 \Delta z$ grid standards.

Lemma 2. If $\theta_0(z) \in L_2(0,H)$ to solve problems (15) - (17) we have estimation:

$$\max_j \left\|\sqrt{C}Y^{j+1}\right\|^2 + \sum_j \left\|\sqrt{\lambda}Y^{j+1}_{i\bar{z}}\right\|^2 \Delta t \le C_1 \sum_i \frac{\Delta z}{\lambda_n(z)}.$$

Here and further C_k, $\mathrm{k} = 1,2,3,\ldots -$ constants, which depend from the input data of the problem.

Lemma 3. If $\theta_0(z) \in L_2(0,H)$ to solve problems (11)-(12) we have estimation

$$\max_j \left\|\sqrt{C}\Delta Y^{j+1}\right\|^2 + \sum_j \left\|\sqrt{\lambda}\Delta Y^{j+1}_{i\bar{z}}\right\|^2 \Delta t \le C_2 \left\|\Delta\lambda\right\|^2,$$

3.3. Iterative scheme

Using (10), we obtain

$$J(\lambda_{n+1}) - J(\lambda_n) = 2\sum_{j=0}^{m-1}\Delta Y_N^{j+1}\left(Y_N^{\;j+1}(\lambda_n) - \theta_g^{j+1}\right)\Delta t + \sum_{j=0}^{m-1}\left(\Delta Y_N^{j+1}\right)^2 \Delta t$$

Using (14) we rewrite it as

$$J(\lambda_{n+1}) - J(\lambda_n) = \sum_{j=0}^{m-1}\left(\Delta Y_N^{j+1}\right)^2 \Delta t - \sum_{j=0}^{m-1}\sum_{i=1}^{N}\Delta\lambda Y^{j+1}_{i\bar{z}}(\lambda_n) U^j_{i\bar{z}} \Delta z \Delta t - \sum_{j=0}^{m-1}\sum_{i=1}^{N}\Delta\lambda\Delta Y^{j+1}_{i\bar{z}} U^j_{i\bar{z}} \Delta z \Delta t$$

If

$$\Delta\lambda = \beta_n \sum_{j=0}^{m-1} Y_{i\bar{z}}^{j+1}(\lambda_n) U_{i\bar{z}}^{j} \Delta t\,,$$

Then

$$J(\lambda_{n+1}) - J(\lambda_n) = \sum_{j=0}^{m-1} \left(\Delta Y_N^{j+1}\right)^2 \Delta t - \sum_{i=1}^{N} \beta_n \left(\sum_{j=0}^{m-1} Y_{i\bar{z}}^{j+1}(\lambda_n) U_{i\bar{z}}^{j} \Delta t \right)^2 \Delta z - \sum_{j=0}^{m-1} \sum_{i=1}^{N} \Delta\lambda \Delta Y_{i\bar{z}}^{j+1} U_{i\bar{z}}^{j} \Delta z \Delta t \quad (18)$$

As the result, an iterative formula was obtained to calculate the thermal conductivity coefficient:

$$\lambda_{n+1}(z) = \lambda_n(z) + \beta_n \sum_{j=0}^{m-1} Y_{i\bar{z}}^{j+1}(\lambda_n) U_{i\bar{z}}^{j} \Delta t \quad (19)$$

Assume that $\lambda(z) -$ piecewise constant function. We introduce notation $h_0 = 0,\ \mathrm{h}_3 = H$ and if $k -$ number of homogeneous subregions, then $\lambda(z) = \lambda(k),\ \mathrm{k} = 1,2,3$. Then, from (19) summing over the variable i in each homogeneous region, we derive a formula

$$\lambda_{n+1}(k) = \lambda_n(k) + \sum_i \frac{\beta_n(k)}{h_k - h_{k-1}} \sum_{j=0}^{m-1} Y_{i\bar{z}}^{j+1}(\lambda_n) U_{i\bar{z}}^{j} \Delta t \Delta z,\ \mathrm{k} = 1\,,2\,,. \quad (20)$$

Theorem 1. If $\theta_0(z) \in L_2(0,H)$ and $0 < C_3(k) \le \lambda_0(k) \le C_4(k) < \infty,\ \mathrm{k} = 1,2,3$ it follows from (20) on the basis of Lemma 1 and 2 controlling parameter $\beta_n(k), \mathrm{k} = 1,2,3$ (18) we obtain the limitation of approximate values of the thermal conductivity coefficient, that is $0 < C_5(k) \le \lambda_n(k) \le C_6(k) < \infty,\ \mathrm{k} = 1,2,3$ for any $n = 1,2,3...$.

Theorem 2. If $\sum_i \frac{\beta_n}{h_k - h_{k-1}} \sum_{j=0}^{m-1} Y_{i\bar{z}}^{j+1}(\lambda_n) U_{i\bar{z}}^{j} \Delta t \Delta z = B_n(k) \ne 0,\ \mathrm{k} = 1,2,3$,, then controlling parameter $\beta_n(k), \mathrm{k} = 1,2,3$, and using Lemma 1.2, and 3 of the formula (20) we obtain monotony of the minimized functional , that is $J(D_{n+1}) - J(D_n) < 0,\ \mathrm{n} = 0,1,2,3,...$.

4. Calculation formulas

1. Set the initial approximation of the thermal conductivity $\lambda_0(k)$, such that

$0 < C_3(k) \le \lambda_0(k) \le C_4(k) < \infty,\ \mathrm{k} = 1,2,3$

2. Solve the direct problem (7) - (9). Calculate approximate values of soil temperature on the earth surface Y_N^{j+1} and difference derivatives Y_{iz}^{j+1}, $\mathrm{i} = 0,1,2,...,\mathrm{N}-1;\ \mathrm{j} = 0,1,2,...,\mathrm{m}-1$.

3. Solve the dual problem (15) - (17). Calculate difference derivatives U_{iz}^{j}, $\mathrm{i} = 0,1,2,...,\mathrm{N}-1;\ \mathrm{j} = m-1, m-2,...,0$.

4. The next approximation of the thermal conductivity calculated by the formula

$$\lambda_{n+1}(k) = \lambda_n(k) + \sum_i \frac{\beta_n(k)}{h_k - h_{k-1}} \sum_{j=0}^{m-1} Y_{i\bar{z}}^{j+1}(\lambda_n) U_{i\bar{z}}^{j} \Delta t \Delta z,\ \mathrm{k} = 1,2,3$$

5. Function $\beta_n(k)$, $\mathrm{k} = 1,2,3$ is chosen sufficiently small so that there is inequality

$$J(\lambda_{n+1}) - J(\lambda_n) < 0.$$

6. If

$$\left| \frac{J(\lambda_{n+1}) - J(\lambda_n)}{J(\lambda_n)} \right| < \varepsilon,$$

then value $\lambda_{n+1}(k)$ is taken as the value $\lambda(k)$ with precision ε.

5. A numerical experiment. To verify the correctness of theoretical conclusions numerical calculations were performed for the three-layered earth: the moist ground- sugar sand-clay. The thickness of each layer, respectively, equal to 20 cm, 20 cm and 100 cm. Layers are arranged in the listed order. Oz- axis directed upward, so layers in figures and in the program located in reverse order. Characteristics of moist ground, sugar sand and clay are taken from the work / 35 /. Numerical values of thermophysical characteristics are given in the table.

Table 1

Name of the material	ρ, kg/m^3	λ, W/(m×Degree)	C, KJ/(kg×Degree)
Clay	1850	1.035	1.089
Sugar sand	1600	0.582	1.256
Moist ground	1700	0.657	2.01

The experiment was conducted when the ambient temperature remains constant for 24 hours. Step to spatial variables $dh = 0.001$м, and the time step $dt = 0.1$hour. During the iteration process, we studied dynamics of the aspiration of approximate values of soil temperature on the earth surface and the thermal conductivity of the soil to the true value.

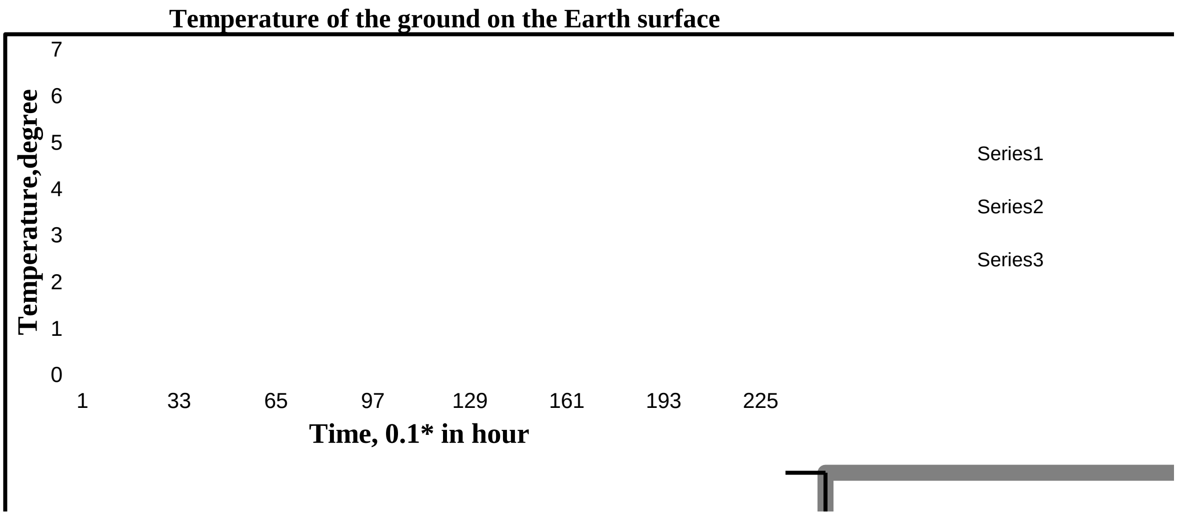

Figure1. Temperature of the soil on the earth surface: first row exact value, second row after 200 iterations, third row after 100 iterations.

Below there is dependence of a functional from number of iterations.

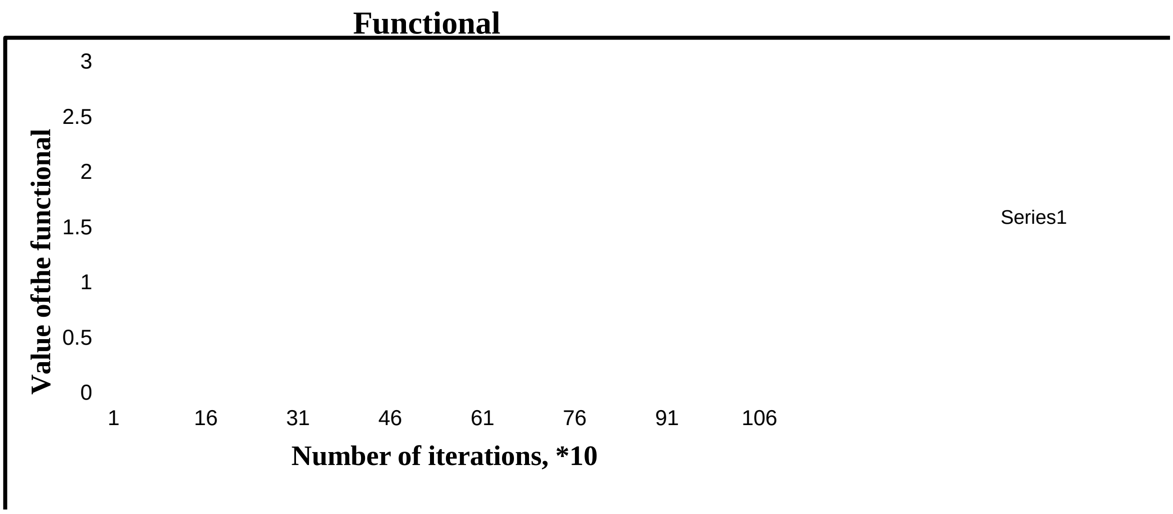

Figure 2. Dinamics of changes in the functional $J(\lambda)$.

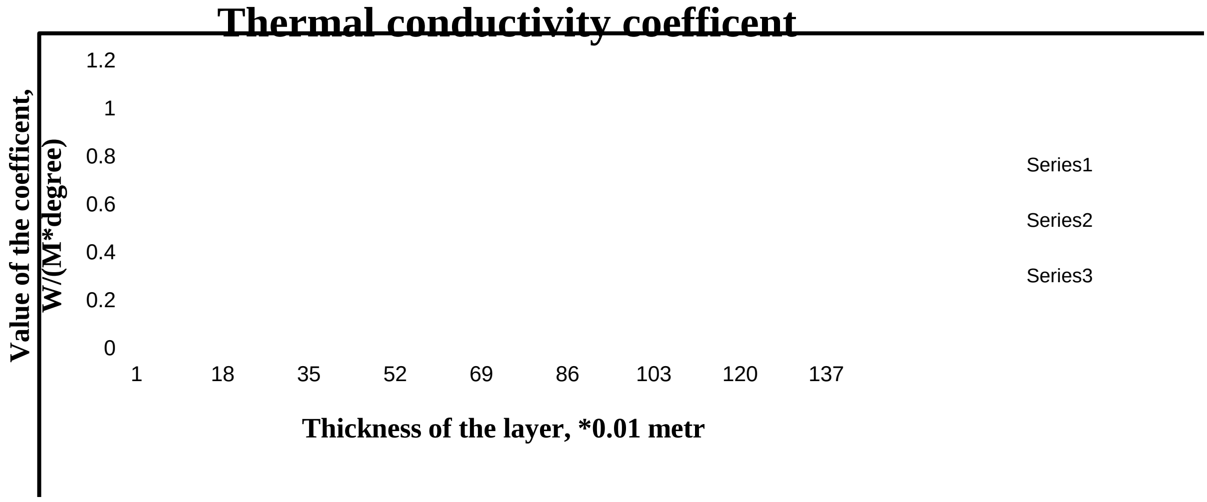

Figure 3. Thermal conductivity coefficient of the soil: first row exact value, second row after 500 iterations, third row after 100 iterations.

Below is a table, containing changes of a maximum error of the method from the number of iterations. From Table 2 it is clear that the error decreases with an increase in the number of iterations.

Table 2

Number of iterations	100	200	300	400	500	1000	1200
Maximum deviation	0.0916	0.0679	0,054	0,0441	0.0364	0,0135	0,0062

Based on the maximum error graph was plotted, which shows dynamics of the error of the proposed iterative method.

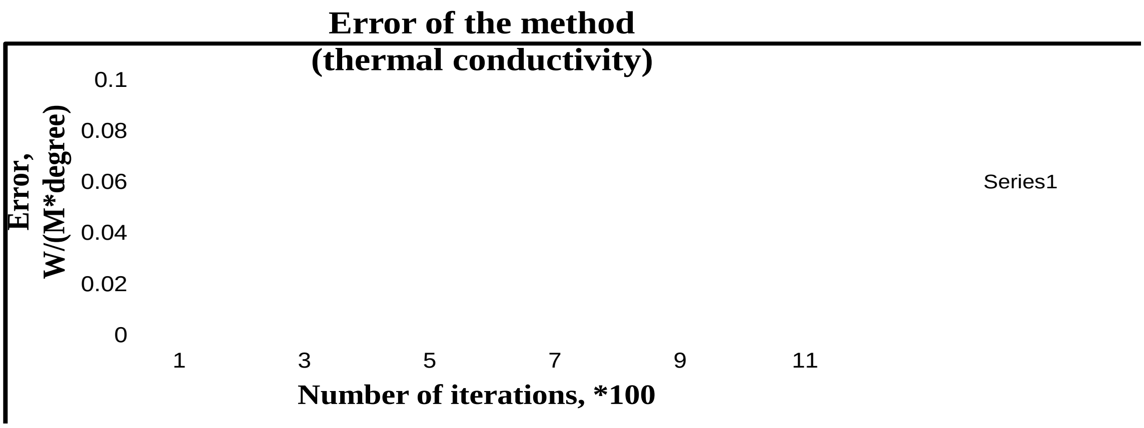

Figure 4. Accuracy of the method. Dynamics of changes in the exact value of the deviation of the thermal conductivity from the approximate value.

Let the ambient temperature varies depending on time. By specifying the day and night temperatures of the air, there was composed interpolation formula

$$T_b(t) = \frac{Tb\max + Tb\min}{2} + \frac{Tb\max - Tb\min}{2}\sin\frac{t\pi}{12}$$

Here $Tb\max$ - daily air temperature, $Tb\min$ - temperature at night. In numerical experiments, we have the following data for air temperature: $Tb\max = 21$ degrees, $Tb\min = 11$ degrees above zero. Step to spatial variables $\Delta z = 0.001$ m and time step $\Delta t = 0.1$ h. Below we provided results of numerical experiments.

The maximum deviation of approximate values of the thermal conductivity from the exact value:

Table 3

Number of iterations	100	200	300	400	500	600	700
Maximum deviation	0.1623	0.1164	0.0893	0.0699	0.0449	0.0426	0.0321

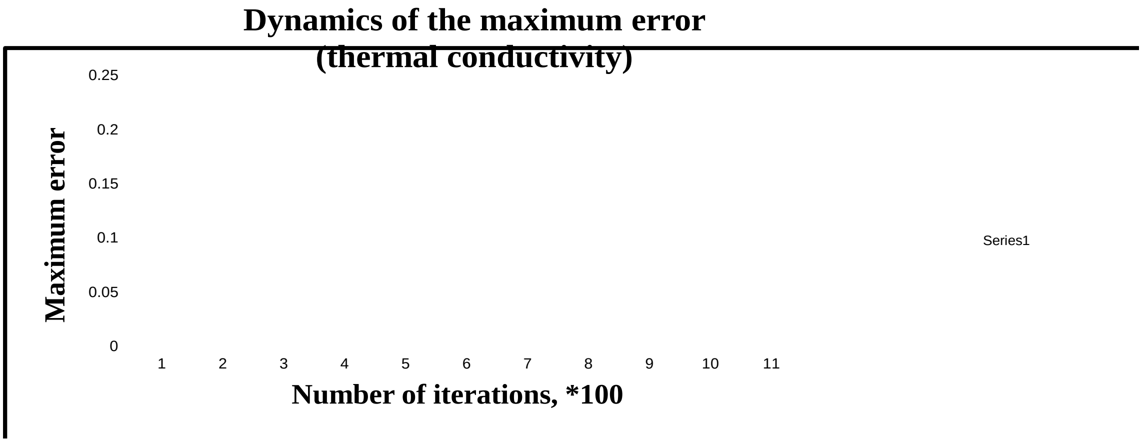

Figure 5. Dependence of the maximum error of the thermal conductivity coefficient on the number of iterations. The air temperature during the day is 21 degrees above zero, and at night 11 degrees above zero.

6. Conclusion.

In this paper the following results were obtained:

1) Conjugate difference problem was made up on the basis of the direct difference problem; 2) An iterative scheme was developed for calculating the thermal conductivity of layered soil;

3) Limitation of approximate values of the thermal conductivity coefficient was proved on the basis of a priori estimates of solutions of direct and conjugate problem ;

4) Managing a sufficiently small parameter achieved a monotone of minimized functional;

5) The numerical calculations confirm the convergence of the iterative method.

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