

# Structural Breaks and Volatility Spillovers: The Case of the US and Canadian Stock Markets

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## Abstract

This paper investigates the relations of structural breaks and volatility spillovers by using the US and Canadian stock return data. Specifically, applying spillover MGARCH models without and with structural break dummy variables to the two stock returns, this study derives the following interesting evidence. (1) First, we reveal that for both the US and Canadian stock returns, the volatility persistence parameter values in our spillover MGARCH models decline when structural break dummy variables are incorporated. (2) Second, we further clarify that when we do not take structural breaks into account, the spillover effect was unidirectional from Canada to the US. However, when we take structural breaks into consideration, the results from our spillover MGARCH model with structural break dummies demonstrate that the volatility spillover effects between the US and Canada become bidirectional. (3) Third, we furthermore reveal that around the Lehman Brothers bankruptcy in 2008, the time-varying volatilities derived from our spillover MGARCH model with structural break dummy variables show slightly higher values than those volatilities from our spillover MGARCH model with no structural break dummy variable.

**Keywords:** MGARCH model, spillover, structural break, volatility persistence

## 1. Introduction

In business and finance literature, structural breaks in time-series stock returns become highly important, and recently, the volatility spillover in international stock markets is also being very important research topic (e.g., Diebold and Yilmaz, 2012; Ewing and Malik, 2016; Tsuji, 2018a; Tsuji, 2018b). Then how are stock return volatility spillovers affected by their structural breaks? Further, how is stock return volatility persistence affected by their structural breaks? In order to answer the above research questions, this study examines how stock return volatility spillovers and volatility persistence are affected by their structural breaks by using the US and Canadian stock return data.

More specifically, this paper quantitatively investigates the relations of structural breaks and volatility spillovers or volatility persistence by using the representative US and Canadian stock index data and applying our spillover MGARCH models without and with structural break dummies to the data. As a result, this study derives the following interesting evidence.

(1) First, we reveal that for both the US and Canadian stock returns, the volatility persistence parameter values in our spillover MGARCH models decline when structural break dummy variables are incorporated. (2) Second, we also clarify that when we do not take structural breaks into account, the spillover effect is unidirectional from Canada to the US. However, when we take structural breaks into consideration, the results from our spillover MGARCH model with structural break dummies suggest that the volatility spillover effects between the US and Canada become bidirectional. (3) Third, we further reveal that around the Lehman crisis in 2008, the time-varying volatilities derived from our spillover MGARCH model with structural break dummy variables show slightly higher values than those volatilities from our spillover MGARCH model with no structural break dummy variable. These interesting findings uncovered in this study represent the important contributions of this article.

As for the rest of this paper, in Section 2, recent related existing research is reviewed, and in Section 3, the data used in this study are documented. After these, in Section 4, the model and methods for our analyses are explained. After that, Section 5 documents our empirical results, and Section 6 summarizes and concludes the paper.

## 2. Related literature review

This section briefly reviews several related extant studies that analyzed structural breaks and/or volatility spillovers. To start, based on MGARCH analyses, Ewing and Malik (2005) suggested that ignoring structural breaks may lead to overestimate the degree of transmission between the conditional variances for stock returns of small and large firms. Using MGARCH models, Sadorsky (2012) analyzed the volatility spillovers between oil futures returns and the stock returns of clean energy and technology firms, and derived some spillover effects among them. Ewing and Malik (2016) examined oil and US stock market returns by an MGARCH model, and showed that after taking structural breaks into account in their model, they found stronger volatility spillover effects between the two markets.

Further, using univariate GARCH models and oil returns, Ewing and Malik (2017) exhibited that incorporating structural breaks into GARCH models reduced the persistence in oil return

volatility, and they concluded that when structural breaks are taken into account, both good and bad news more significantly affected oil return volatility. Moreover, by applying a new DCC-MEGARCH model, Tsuji (2018c) explored return and volatility transmission between international oil equities and WTI crude oil futures, in which structural break analyses were also conducted for robustness checks. Furthermore, Tsuji (2018a, 2018b) also investigated the linkages between stock return structural breaks and their volatility persistence, although the examinations were by univariate analyses.

As above, we understand that in recent business, economics, and finance research, analyzing structural breaks and/or volatility spillovers by applying GARCH techniques becomes highly important. Therefore, in this paper, by using the US and Canadian stock return data, we investigate the effect of structural breaks on volatility spillovers in the framework of MGARCH analyses.

### 3. Data

In this section, we explain our data and variables for this study. All raw stock price data are from Thomson Reuters. More concretely, our first variable is the daily log difference percentage stock return computed using the US S&P 500, which we denote as LRUS. Our second variable is the daily log difference percentage stock return computed using the Canadian Toronto stock exchange (TSX) composite index, which we denote as LRCAN. The sample period of these two returns for this study is from January 4, 2000 to August 3, 2018.

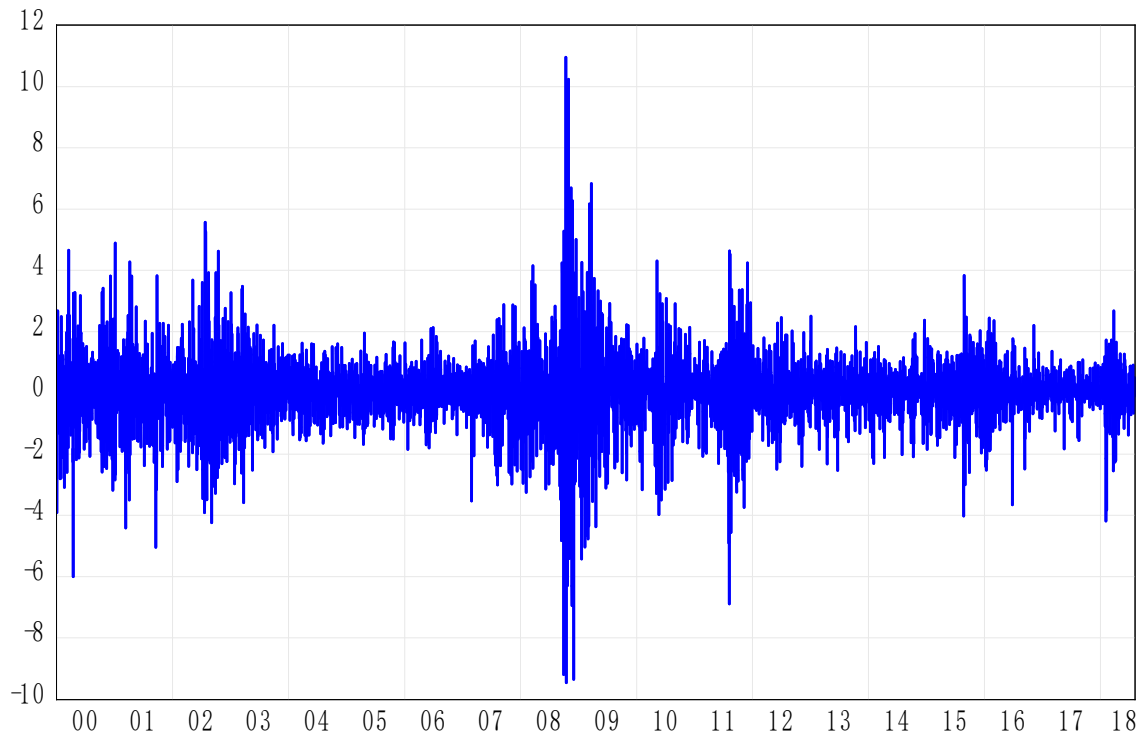
Figure 1 plots the time-series evolution of the above daily log S&P 500 and TSX composite index returns (Panels A and B, respectively) for the period from January 4, 2000 to August 3, 2018. Further, Table 1 provides the summary statistics regarding the US and Canadian stock returns. As Table 1 shows, for both two stock index return series for the US and Canada, their mean values take slightly positive values and their skewness values present negative values. In addition, from Table 1, it is also understood that their kurtosis values are much higher than the kurtosis value of normal distributions.

Table 1. Summary statistics as to the daily log stock returns for the US and Canada

Statistic	LRUS	LRCAN
Mean	0.0138	0.0138
Median	0.0240	0.0368
Maximum	10.9572	9.3703
Minimum	-9.4695	-9.7880
Standard deviation	1.1854	1.0746
Skewness	-0.2244	-0.6732
Excess Kurtosis	9.0742	10.2904

Notes: Our full sample period is from January 4, 2000 to August 3, 2018, with 4849 observations for the two returns.

Panel A. LRUS



Panel B. LRCAN

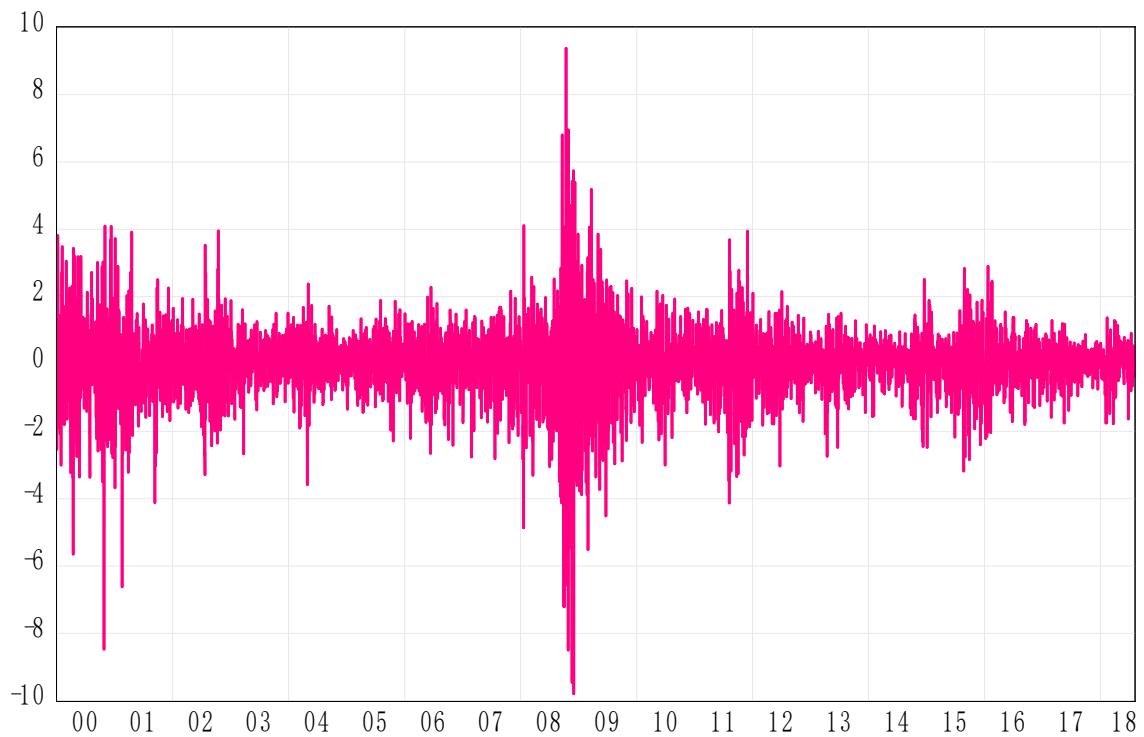


Figure 1. Daily percentage log stock return evolution for the US and Canada

#### 4. Models and methods

We next describe our analyzing methods. In this study, as documented, we use the MGARCH model incorporating spillover terms for our analyses. That is, for the US and Canadian stock returns, LRUS and LRCAN, we estimate the spillover MGARCH models without structural break dummy variable as the following equations (1) to (4) and the spillover MGARCH models with structural break dummy variables as the following equations (1), (2), (5) and (6).

$$R_{1,t} = \mu_1 + \varepsilon_{1,t}, \quad (1)$$

$$R_{2,t} = \mu_2 + \varepsilon_{2,t}, \quad (2)$$

$$\sigma_{1,t}^2 = c_1 + \alpha_{1,1}\varepsilon_{1,t-1}^2 + \alpha_{1,2}\varepsilon_{2,t-1}^2 + \beta_1\sigma_{1,t-1}^2, \quad (3)$$

$$\sigma_{2,t}^2 = c_2 + \alpha_{2,2}\varepsilon_{2,t-1}^2 + \alpha_{2,1}\varepsilon_{1,t-1}^2 + \beta_2\sigma_{2,t-1}^2, \quad (4)$$

$$\sigma_{1,t}^2 = c_1 + \alpha_{1,1}\varepsilon_{1,t-1}^2 + \alpha_{1,2}\varepsilon_{2,t-1}^2 + \beta_1\sigma_{1,t-1}^2 + \sum_{i=1}^{12} d_{i,1,1} USD_{i,t} + \sum_{j=1}^{16} d_{j,1,2} CAND_{j,t}, \quad (5)$$

$$\sigma_{2,t}^2 = c_2 + \alpha_{2,2}\varepsilon_{2,t-1}^2 + \alpha_{2,1}\varepsilon_{1,t-1}^2 + \beta_2\sigma_{2,t-1}^2 + \sum_{k=1}^{12} d_{k,2,1} USD_{k,t} + \sum_{l=1}^{16} d_{l,2,2} CAND_{l,t}. \quad (6)$$

In equations (1) and (2),  $R_{1,t}$  means the US log stock return, LRUS; and  $R_{2,t}$  means the Canadian log stock return, LRCAN. Further,  $\varepsilon_{1,t}$  means the error term of the US stock return; while  $\varepsilon_{2,t}$  means the error term of the Canadian stock return. Moreover, in equations (3) to (6),  $\sigma_{1,t}$  means the volatility of the US log stock return; while  $\sigma_{2,t}$  means the volatility of the Canadian log stock return. Further,  $\alpha_{1,1}$  and  $\alpha_{2,2}$  denote the ARCH parameters for the US and Canadian stock returns, respectively. In addition,  $\alpha_{1,2}$  denotes the parameter of volatility spillovers from the Canadian stock return to the US stock return; and  $\alpha_{2,1}$  denotes the parameter of volatility spillovers from the US stock return to the Canadian stock return. Furthermore,  $\beta_1$  denotes the GARCH parameter for the US stock return volatility; and  $\beta_2$  denotes the GARCH parameter for the Canadian stock return volatility.

Explaining our dummy variable construction procedures, after identifying the structural break points for each of LRUS and LRCAN by employing the iterated cumulative sums of squares (ICSS) algorithm, this study constructs the dummy variables reflecting structural breaks for the two return series. Table 2 presents the number of the determined points of structural breaks and the time periods for the two return series. As in Table 2, for our full sample period, we identify 12 structural break points for LRUS and 16 break points for LRCAN. In this paper, as shown in equations (5) and (6), the structural break dummy variables for LRUS are denoted as  $USD_i$ ; while those for LRCAN are denoted as  $CAND_i$ .

Illustrating these dummy variables,  $USD_{1,t}$  takes one for the period from January 4, 2000 through to June 14, 2002, and zero elsewhere; and  $USD_{2,t}$  takes one for the period from June 17, 2002 through to October 17, 2002, and zero elsewhere. Similarly,  $CAND_{1,t}$  takes one for the period from January 4, 2000 through to April 18, 2001, and zero elsewhere; and  $CAND_{2,t}$  takes one for the period from April 19, 2001 through to July 9, 2002, and zero elsewhere.

Table 2. Structural break identifications for the US and Canadian stock returns

Series	Break point	Time period
LRUS	12	January 4, 2000 – June 14, 2002
		June 17, 2002 – October 17, 2002
		October 18, 2002 – April 28, 2003
		April 29, 2003 – May 11, 2004
		May 12, 2004 – July 9, 2007
		July 10, 2007 – September 12, 2008
		September 15, 2008 – December 2, 2008
		December 3, 2008 – May 18, 2009
		May 19, 2009 – September 3, 2010
		September 6, 2010 – August 1, 2011
		August 2, 2011 – December 20, 2011
		December 21, 2011 – June 30, 2016
		July 1, 2016 – August 3, 2018
LRCAN	16	January 4, 2000 – April 18, 2001
		April 19, 2001 – July 9, 2002
		July 10, 2002 – December 18, 2002
		December 19, 2002 – October 3, 2005
		October 4, 2005 – July 23, 2007
		July 24, 2007 – September 1, 2008
		September 2, 2008 – December 1, 2008
		December 2, 2008 – June 25, 2009
		June 26, 2009 – December 4, 2009
		December 7, 2009 – July 25, 2011
		July 26, 2011 – January 3, 2012
		January 4, 2012 – July 11, 2013
		July 12, 2013 – September 18, 2014
		September 19, 2014 – February 3, 2015
		February 4, 2015 – August 18, 2015
August 19, 2015 – February 19, 2016		
February 22, 2016 – August 3, 2018		

Notes: The sample period for our analyses is from January 4, 2000 through to August 3, 2018, and the usable return observations is 4849. The ICSS algorithm is used for identifying the structural break points.

## 5. Empirical results

This section documents our empirical results. First, Table 3 presents the estimation results of our spillover MGARCH models for LRUS and LRCAN with no structural break dummy variable (equations (1) to (4)). Next, Table 4 presents the estimation results of our spillover MGARCH models for LRUS and LRCAN with structural break dummy variables (equations (1), (2), (5) and (6)), and in Table 4,  $d$  means the parameters of our dummy variables.

Table 3. Estimation results of the spillover MGARCH model without structural break dummy variable for the US and Canadian stock returns

Mean equation				
Parameter	Estimates	Standard error	<i>t</i> -statistic	<i>p</i> -value
$\mu_1$	0.0479***	0.0113	4.2537	0.0000
$\mu_2$	0.0454***	0.0102	4.4651	0.0000
Variance equation				
Parameter	Estimates	Standard error	<i>t</i> -statistic	<i>p</i> -value
$c_1$	0.0135***	0.0024	5.6784	0.0000
$c_2$	0.0062***	0.0016	3.9142	0.0001
$\alpha_{1,1}$	0.0827***	0.0076	10.8258	0.0000
$\alpha_{1,2}$	0.0285***	0.0060	4.7462	0.0000
$\alpha_{2,1}$	0.0029	0.0027	1.0785	0.2808
$\alpha_{2,2}$	0.0749***	0.0076	9.9119	0.0000
$\beta_1$	0.8832***	0.0091	97.1127	0.0000
$\beta_2$	0.9162***	0.0083	110.5483	0.0000
LL	-12554.7448			

Notes: The sample period for our analyses is from January 4, 2000 through to August 3, 2018. The number of the usable return observations is 4849. LL denotes the log-likelihood value.

Comparing the results shown in Tables 3 and 4, the following is evident. First, for LRUS, the GARCH parameter (volatility persistence parameter) values of the spillover MGARCH model ( $\beta_1$ ) clearly decrease from 0.8832 (Table 3) to 0.8120 (Table 4) when our structural break dummies are included. Likewise, for LRCAN, the GARCH parameter (volatility persistence parameter) values of the spillover MGARCH model ( $\beta_2$ ) also largely decrease from 0.9162 (Table 3) to 0.7767 (Table 4) when our structural break dummies are included. Hence, our estimation results of the spillover MGARCH models with and without the dummy variables for structural breaks indicate that when we consider stock return structural breaks, stock return volatility persistence clearly declines. That is, when we do not take structural



breaks in stock returns into consideration, volatility persistence in stock returns is overestimated in MGARCH models.

We next explain the estimation results of the spillover parameters in our models. Comparing the results shown in Tables 3 and 4, we understand the following. First, the spillover parameters from Canada to the US in our spillover MGARCH model with and without structural break dummies ( $\alpha_{1,2}$ ) are statistically significantly positive both in Tables 3 and 4. Second, the spillover parameter from the US to Canada in our spillover MGARCH model without structural break dummy ( $\alpha_{2,1}$ ) is not statistically significant in Table 3. However, when we consider stock return structural breaks, in Table 4, the spillover parameter from the US to Canada in our spillover MGARCH model with structural break dummies ( $\alpha_{2,1}$ ) becomes statistically significantly positive at the 5% level.

That is, when we do not take structural breaks into account, the spillover effect is unidirectional from Canada to the US. However, when we consider structural breaks, our estimation results of our spillover MGARCH model with structural break dummies clarify that the volatility spillover effects between the US and Canada become bidirectional.

Table 4. Estimation results of the spillover MGARCH model with structural break dummies for the US and Canadian stock returns

Mean equation				
Parameter	Estimates	Standard error	<i>t</i> -statistic	<i>p</i> -value
$\mu_1$	0.0536***	0.0110	4.8540	0.0000
$\mu_2$	0.0448***	0.0104	4.3113	0.0000
Variance equation				
Parameter	Estimates	Standard error	<i>t</i> -statistic	<i>p</i> -value
$c_1$	0.0309***	0.0060	5.1776	0.0000
$c_2$	0.0357***	0.0072	4.9575	0.0000
$\alpha_{1,1}$	0.0642***	0.0090	7.1533	0.0000
$\alpha_{1,2}$	0.0232**	0.0093	2.4961	0.0126
$\alpha_{2,1}$	0.0146**	0.0065	2.2592	0.0239
$\alpha_{2,2}$	0.0543***	0.0100	5.4149	0.0000
$\beta_1$	0.8120***	0.0216	37.6094	0.0000
$\beta_2$	0.7767***	0.0305	25.4853	0.0000
$d_{1,1,1}$	-15.1882***	5.4768	-2.7732	0.0056
$d_{1,2,1}$	-16.4603***	5.9137	-2.7834	0.0054
$d_{2,1,1}$	-14.7561***	5.4733	-2.6960	0.0070



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$d_{2,2,1}$	-16.4162***	5.9116	-2.7770	0.0055
$d_{3,1,1}$	-15.1518***	5.4661	-2.7720	0.0056
$d_{3,2,1}$	-16.5507***	5.9129	-2.7991	0.0051
$d_{4,1,1}$	-15.3230***	5.4662	-2.8032	0.0051
$d_{4,2,1}$	-16.5460***	5.9125	-2.7985	0.0051
$d_{5,1,1}$	-15.3492***	5.4658	-2.8083	0.0050
$d_{5,2,1}$	-16.5562***	5.9125	-2.8002	0.0051
$d_{6,1,1}$	-15.0835***	5.4687	-2.7582	0.0058
$d_{6,2,1}$	-16.3439***	5.9094	-2.7657	0.0057
$d_{7,1,1}$	-12.9326**	5.2733	-2.4525	0.0142
$d_{7,2,1}$	-14.6421**	5.7212	-2.5593	0.0105
$d_{8,1,1}$	3.7552	2.5071	1.4978	0.1342
$d_{8,2,1}$	1.4644	1.0443	1.4023	0.1608
$d_{9,1,1}$	3.3663	2.4949	1.3493	0.1773
$d_{9,2,1}$	1.3065	1.0231	1.2771	0.2016
$d_{10,1,1}$	3.2969	2.4927	1.3226	0.1860
$d_{10,2,1}$	1.2872	1.0233	1.2579	0.2084
$d_{11,1,1}$	0.4269***	0.1301	3.2824	0.0010
$d_{11,2,1}$	0.2297**	0.1123	2.0461	0.0407
$d_{12,1,1}$	0.0351**	0.0159	2.2063	0.0274
$d_{12,2,1}$	0.0278**	0.0138	2.0148	0.0439
$d_{1,1,2}$	15.3458***	5.4746	2.8031	0.0051
$d_{1,2,2}$	16.8625***	5.9165	2.8501	0.0044
$d_{2,1,2}$	15.3068***	5.4755	2.7955	0.0052
$d_{2,2,2}$	16.5314***	5.9147	2.7950	0.0052
$d_{3,1,2}$	15.3164***	5.4650	2.8027	0.0051
$d_{3,2,2}$	16.6468***	5.9138	2.8149	0.0049
$d_{4,1,2}$	15.3634***	5.4656	2.8109	0.0049
$d_{4,2,2}$	16.5744***	5.9127	2.8032	0.0051
$d_{5,1,2}$	15.3611***	5.4657	2.8105	0.0049
$d_{5,2,2}$	16.6189***	5.9134	2.8104	0.0049

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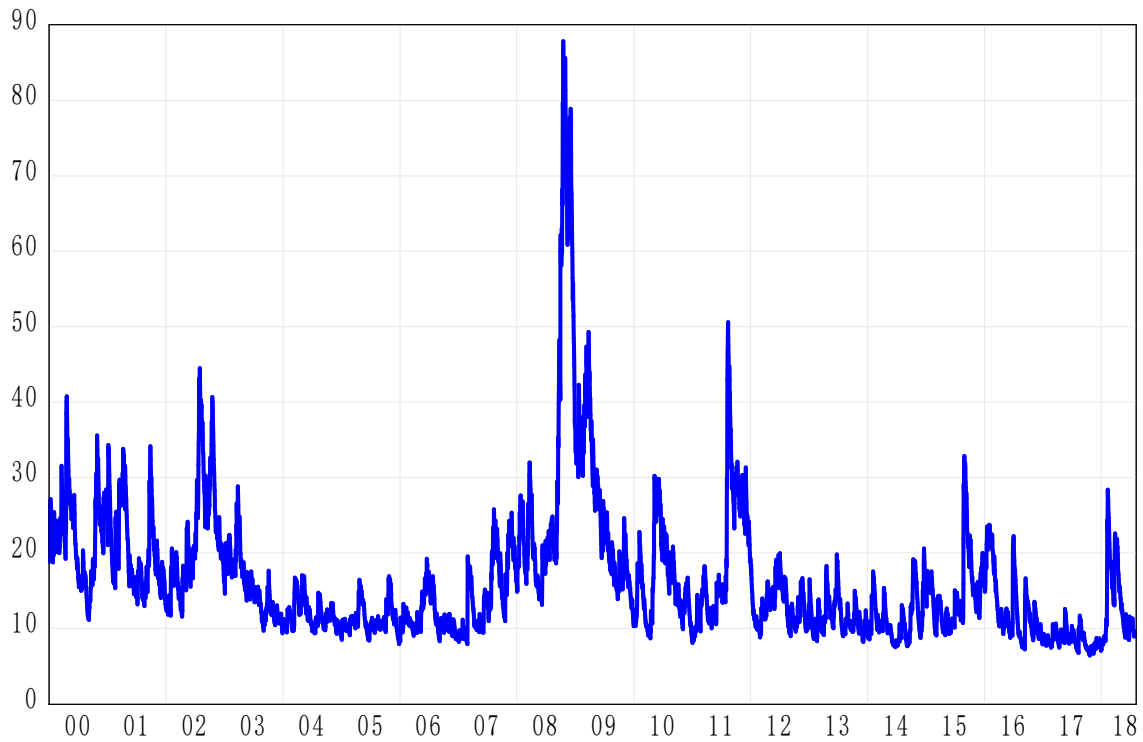
$d_{6,1,2}$	15.2352***	5.4665	2.7870	0.0053
$d_{6,2,2}$	16.5019***	5.9122	2.7912	0.0053
$d_{7,1,2}$	15.9053***	5.4725	2.9064	0.0037
$d_{7,2,2}$	18.0749***	5.9307	3.0477	0.0023
$d_{8,1,2}$	-3.1982	2.4950	-1.2818	0.1999
$d_{8,2,2}$	-0.8037	1.0475	-0.7673	0.4429
$d_{9,1,2}$	-3.2802	2.4938	-1.3153	0.1884
$d_{9,2,2}$	-1.1270	1.0270	-1.0974	0.2725
$d_{10,1,2}$	-3.2599	2.4925	-1.3079	0.1909
$d_{10,2,2}$	-1.2460	1.0241	-1.2167	0.2237
$d_{11,1,2}$	-0.0996	0.0623	-1.5985	0.1099
$d_{11,2,2}$	0.0484	0.0871	0.5563	0.5780
$d_{12,1,2}$	-0.0034	0.0168	-0.2021	0.8398
$d_{12,2,2}$	0.0104	0.0151	0.6902	0.4901
$d_{13,1,2}$	-0.0238	0.0162	-1.4700	0.1416
$d_{13,2,2}$	-0.0305**	0.0142	-2.1419	0.0322
$d_{14,1,2}$	0.0031	0.0240	0.1300	0.8966
$d_{14,2,2}$	0.0911***	0.0337	2.7053	0.0068
$d_{15,1,2}$	-0.0108	0.0181	-0.5953	0.5517
$d_{15,2,2}$	-0.0022	0.0173	-0.1280	0.8981
$d_{16,1,2}$	0.0828**	0.0405	2.0444	0.0409
$d_{16,2,2}$	0.1633***	0.0495	3.2977	0.0010
LL	-12361.0027			

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Notes: The sample period for our analyses is from January 4, 2000 through to August 3, 2018. The number of the usable return observations is 4849. LL denotes the log-likelihood value.

We also compute the time-varying volatilities using the US and Canadian stock returns and our spillover MGARCH models with and without the dummy variables for stock return structural breaks. Figure 2 displays the time-series evolution of the daily annualized volatilities of the US (Panel A) and Canadian (Panel B) stock returns from our spillover MGARCH model without structural break dummy variable. On the other hand, Figure 3 displays those volatilities of the US (Panel A) and Canadian (Panel B) stock returns from our spillover MGARCH model that incorporates structural break dummies.

Panel A. US



Panel B. Canada

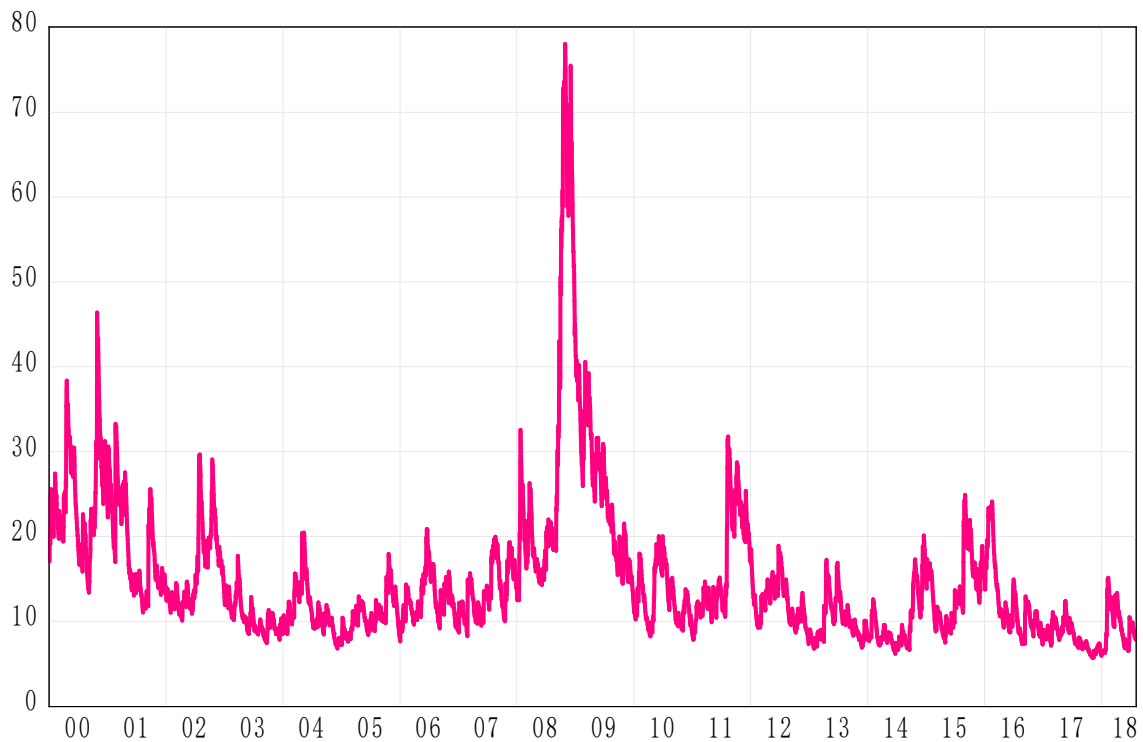
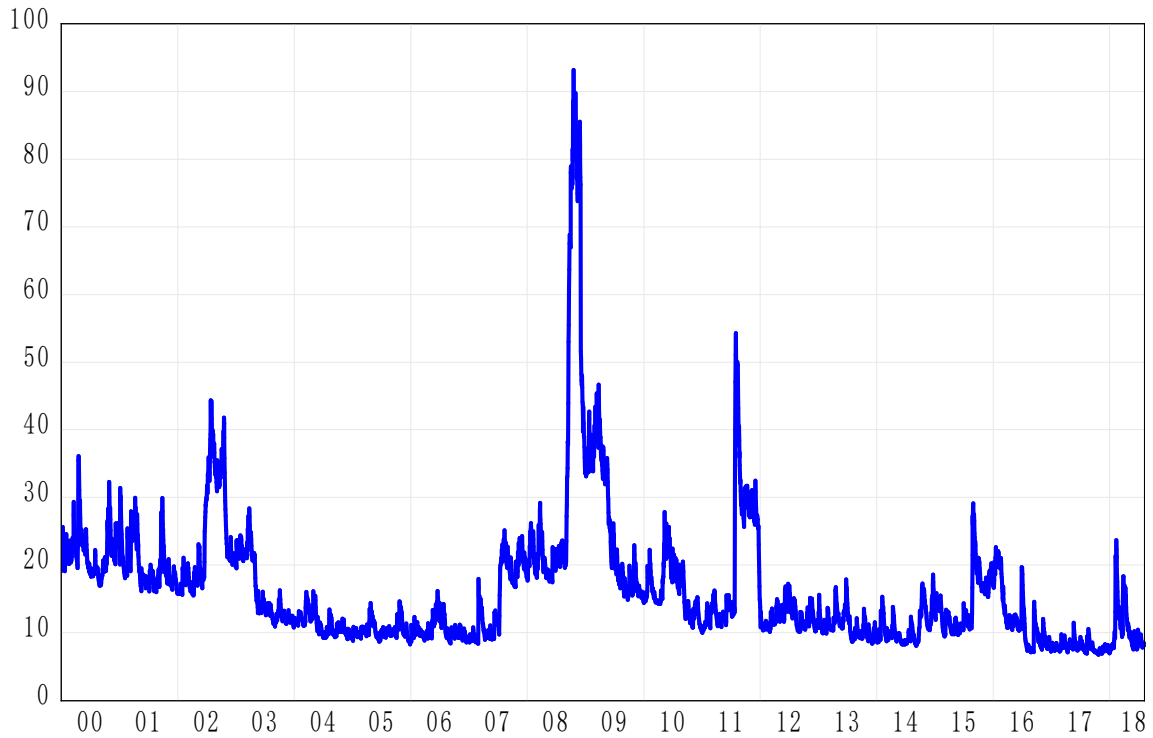


Figure 2. Daily evolution of the time-varying volatilities of the US and Canadian stock returns from the no structural break spillover MGARCH model

Panel A. US



Panel B. Canada

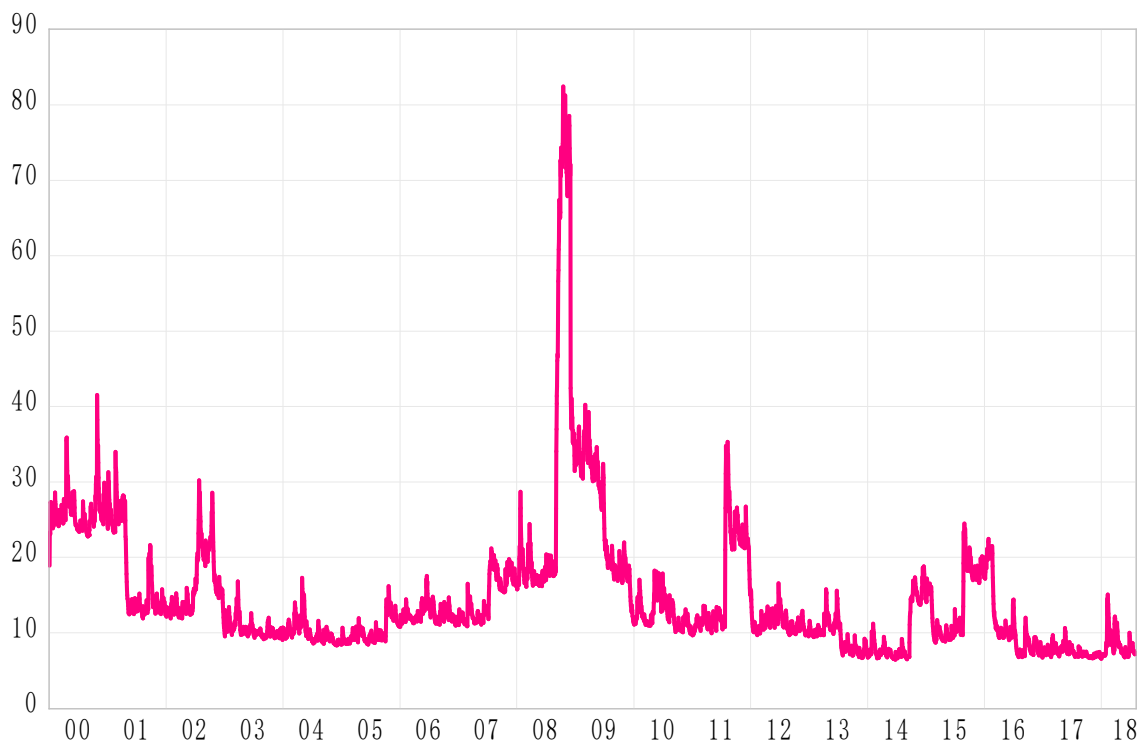


Figure 3. Daily evolution of the time-varying volatilities of the US and Canadian stock returns from the spillover MGARCH model with structural breaks

As in Figures 2 and 3, the estimated time-varying volatilities are generally similar; however, from Figures 2 and 3, we also understand that around the Lehman Brothers bankruptcy in 2008, both for the US and Canadian stock returns, the time-varying volatilities derived from our spillover MGARCH model that incorporates structural break dummy variables are slightly higher than those derived from our spillover MGARCH model with no structural break dummy variable. We consider that this is because in the model with structural break dummies, as documented, volatility persistence decrease; and thus, the volatilities both for the US and Canadian stock returns, which are derived from the model with structural break dummies, sharply increase during the US Lehman crisis in 2008.

## **6. Summary, implications, and Conclusions**

This paper quantitatively investigated the relations of structural breaks and volatility spillovers by using the stock return data of the US S&P 500 and Canadian Toronto stock exchange composite index. In the fields of business and finance, it is well-known that GARCH approach is highly useful and beneficial as Bollerslev (1986, 1990), Nelson (1991), Glosten et al. (1993), Engle and Kroner (1995), Tsuji (2016, 2017a, 2017b), and many other extant studies signified. Based on this, applying the spillover MGARCH models without and with structural break dummy variables, this study derived the following interesting evidence.

(1) First, we revealed that for both the US and Canadian stock returns, the GARCH parameter values, i.e., the volatility persistence parameter values in our spillover MGARCH models clearly declined when the structural break dummy variables are incorporated into our model. This result is for instance consistent with those shown in Ewing and Malik (2016, 2017).

(2) Second, we further clarified that when we do not take structural breaks into account, the spillover effect was unidirectional from Canada to the US. However, when we take structural breaks into consideration, the results from our spillover MGARCH model with structural break dummies suggested that the volatility spillover effects between the US and Canada were bidirectional.

(3) Third, we furthermore revealed that the time-varying volatilities derived from our spillover MGARCH models without and with structural break dummy variables were generally similar. However, it is noteworthy that around the Lehman Brothers bankruptcy in 2008, the time-varying volatilities derived from our spillover MGARCH model with structural break dummies showed slightly higher values than those from our spillover MGARCH model with no structural break dummy.

To sum up, when we do not take structural breaks in stock returns into consideration, in GARCH models, stock return volatility persistence may be overestimated. In addition, when we consider stock return structural breaks, the derived values of the time-varying volatilities changed. Moreover, our results also showed that when we consider volatility spillover effects in international stock markets, it is highly important to take stock return structural breaks into account.

Furthermore, adding managerial implications finally, we first emphasize that our results suggested the importance of estimating asset return volatilities accurately as also suggested

by Tsuji (2016). More specifically, from the viewpoint of downside risk management in all firms in the world, asset return volatilities are highly crucial because volatility is a key parameter of computing, for instance, the Value at Risk, which is one of the most significant measures of downside risk for all firms. Relating to this, Tsuji (2018d) suggested that downside risk is highly important also in considering volatility spillovers since volatility spillovers are often combined with the well-known leverage effect. Further, Tsuji (2018d) also suggested that the mutual asymmetric volatility spillovers found in the study can be interpreted as the bidirectional spillovers of downside risk in international asset markets. Therefore, for all firms, in order to manage their downside risk appropriately, we stress that estimating asset return volatilities very accurately by taking the spillover effects and the structural breaks in asset returns into consideration is highly important as our current study demonstrated.

We believe that the time-series modeling exhibited in this study is widely applicable to many other time-series data in business and finance. Therefore, further detailed research by using other multivariate time-series data with taking volatility spillovers and structural breaks into consideration is one of our future works.

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