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Endogenous Tradability, International Relative Prices, and the International Transmission of Business Cycles

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Abstract

This paper propose a two-country, dynamic, stochastic, general equilibrium (DSGE) model with endogenous tradability, product differentiation, variously determined physical capital, and an elastic labor supply to explore the propagation of business cycles across countries. The model successfully addresses international relative price dynamics (its appreciation with positive home productivity shock, called the 'Harrod-Balassa-Samuelson Effect') through the entry of producers and their cut-off productivities of exporting. The use of endogenous physical capital in the model induces a more realistic framework since the simulated model is compared to the U.S. investment data that covers spending on capital equipment, structures and inventories for producers' entry and exit dynamics. Building the model with endogenous capital and elastic labor supply weakens the volatility of investment compared to conventional international real business cycle (IRBC) models. The model also accounts for several features of the data, such as the volatility of aggregate variables and their correlations with GDP.

Keywords: Heterogeneous firms, International real business cycles, Terms of trade

JEL Classification: E32, F41, F44

1. Introduction

Conventional international real business cycle models (IRBC) fail to account for the appreciation of the international relative prices (e.g. the real exchange rate or the terms of trade) after an aggregate positive productivity shock in the domestic economy. In standard IRBC models, as in Backus, Kehoe & Kydland (1992, 1994), the relative price of domestically produced goods decreases when domestic GDP increases in response to a positive productivity shock. This leads to an international relative prices depreciation and, as a result, the real exchange rate is positively linked to the ratio of consumption across the two economies. If consumption is cheap in the economy due to increase in wealth, then households consume more (efficient risk sharing). However, the empirical findings show that the real exchange rate or terms of trade appreciate with a domestic positive productivity shock and that the correlation between relative consumption across countries and the terms of trade is negative or close to zero. My main contribution in this paper is to construct a two-country, dynamic, stochastic, general equilibrium (DSGE) model with endogenous tradability and heterogenous firms that explains the dynamics of international relative price appreciation and closes for this gap between the model and the data. I believe that the endogenous tradability and heterogeneity in individual firm productivities are the key structure that encourage the dynamics of international relative prices along the international propagation of business cycles.

Based on a model of firms' entry and exit process with heterogeneity in productivities, I augment the models of Bergin & Glick (2007) and Ghironi & Melitz (2005) to include variable physical capital and an elastic supply of labor. The use of endogenous physical capital in the model fosters a more realistic framework since the simulated model is compared to U.S. investment data that covers spending on capital equipment, structures and inventories for producers â entry and exit dynamics. The model also incorporates a sunk entry cost and two types of trade costs that affect the decision of intermediate goods producers. One is the form of ice-berg trade costs and the other is the fixed cost of international trade(note 1). Both Bergin & Glick (2007) and Ghironi & Melitz (2005) analyze the Harrod-Balassa-Samuelson effect(note 2) using endogenous tradability with heterogenous firm-specific productivity, but Bergin & Glick (2007) do not consider aggregate productivity shocks and instead added a heterogeneity of trade cost. Another difference with Bergin & Glick (2007) and Ghironi & Melitz (2005) is that the formal model has cross sector trade, while the later one has within sector trade. Both models use labor as the only input and it is fixed. For the same reason to add variously determined physical capital, augmenting the model with elastic labor brings a more practical framework. Alessandria & Choi (2007) study whether the sunk costs of exporting matter along business cycles. They conclude that entry costs only matter for firm-level dynamics, having little effect on aggregate fluctuations. They use endogenous labor and capital as inputs, but they do not consider the entry process and the fraction of exporters are constant.

The model is composed of a representative consumer, perfectly competitive final goods producers, and monopolistically competitive, heterogeneous intermediate goods producers.

Each intermediate goods producer differs in their productivity, but face a common aggregate productivity shock. I study this model under an incomplete asset market as a baseline model, allowing for the international trade of state non-contingent bonds and limited risk sharing. I find that there are two effects that move the international relative prices in different direction. One is the GDP effect that induces the terms of trade to increase (depreciate) with a home positive productivity shock; the other is the factor efficiency effect which induces to decrease (appreciate). Both effects happen through the relative average exporting cut-off productivity. The GDP at home increase when there is an aggregate productivity shock. This induces the demands for varieties to increase at home so that even less productive foreign firms can enter the home market to fulfill their demands. However, there is the factor efficiency effect that moves the relative average cut-off productivity against the GDP effect. With a domestic productivity shock, factor production costs decrease and bring more firms to enter the home market. More entry in the home market generates competition among firms and this only lets more productive foreign firms enter the home market. It means that the foreign cutoff productivity increases with an aggregate productivity shock while the relative cut-off productivity decrease, as do the import prices relative to producer prices.

There are continual advancements in research finding solutions to the international relative prices anomaly. Backus & Smith (1993) were the first to note this lack of agreement between theoretical studies in which the real exchange rate or the terms of trade are positively correlated with the ratio of consumption across countries, but negative or close to zero in the data. Ghironi & Melitz (2005) show clear endogenous appreciation of the real exchange rate, but they only analyze the long-run effect of appreciation. The quantitative analysis of the benchmark model with endogenous capital accumulation and supply of labor reflects empirical evidence even in the short run along the IRBC. Corsetti, Martin & Pesenti (2007) build a model with trade costs and product variety; then, find terms of trade appreciation when the entry cost is reduced. Chari, Kehoe & McGrattan (2002) find that the volatility of the terms of trade is generated by monetary shocks interacting with sticky goods prices. However, their model is unsuccessful to generate the real exchange rate dynamics. Tuesta (2013), Corsetti, Dedola & Leduc (2008), and Benigno & Thoenissen (2008) show that introducing non-traded goods helps to reconcile theory with the data on terms of trade appreciation and volatility with a domestic productivity shock. Corsetti et al. (2008) argue that the low elasticity of substitution between home and foreign goods with incomplete asset market can help to solve the international relative prices anomaly because productivity shock generate enough wealth effect so that a home productivity shock increase the price of home produced goods. Recently, Moon (2016) examine the terms of trade appreciation with a positive productivity shock in a DSGE model with staggered price setting in non-tradable sector and international trade in intermediate goods sector. Nam & Wang (2010) provided an analysis in which news shock to total factor productivity (TFP) with variable capital utilization generate appreciation of the terms of trade and the real exchange rate.

From a quantitative analysis, three major findings are revealed. First, the quantitative analysis reflects the dynamics of international relative prices, as seen in empirical literature in which it appreciates after an aggregate productivity shock. The factor efficiency effect is bigger than the income effect because of the risk sharing between countries. Due to a reduction in factor production costs, more firms enter into the home economy and generate more competition while the terms of efficiency decrease. Only more productive foreign firms can enter the market due to the higher competition among firms. The simulated model supports these dynamics. Finally, the terms of trade appreciate while relative consumption increases. Thus, the model accounts for the negative co-movement observed in the data between the terms of trade and relative consumption across countries.

Second, the model accounts for several other features of the data such as the volatility of aggregate variables and their correlations with GDP, but it does not replicate the cross-country co-movement due to strong international production shifting. The benchmark model generates the volatility of output, consumption, investment and employment similar to that found in the data. In addition, their correlations with GDP are all positive. However, observed cross-country output, investment and employment co-movements are negative, while cross-country consumption is positive as in standard IRBC models. The quantity anomaly, as brought up by Backus et al. (1992), states that cross-country consumption correlations are generally similar to or lower than cross-country output correlations in the data, whereas standard IRBC models typically produce much higher consumption correlations than output correlations. As in standard IRBC models, trade in assets leads to capital flow and induces negative business cycle correlations between countries. Therefore, the quantity anomaly still exists in the benchmark model.

Third, augmenting the model with endogenous capital and labor undermines the volatility of investment compared with standard IRBC models, as well as the volatility of entry compared to Ghironi & Melitz (2005). Investment in the baseline model is still the most volatile, but with a reduced variability when contrasted with conventional IRBC models. There is a common consensus that the volatility of investment is reduced if capital adjustment costs are accounted. Incorporating capital adjustment cost increases the relative price of capital goods and the agents smoothe their consumption. In my model, extensive margin works similar to capital adjustment cost. The entry condition is determined in terms of effective labor, so it uses labor more than capital goods. Therefore, even without capital adjustment costs, the volatility of investment can be reduced. The volatility of entry is much smaller compared to the no-capital, fixed labor economy of Ghironi & Melitz (2005). This is due to asset market friction in which each country attempts to create more balanced trade since home agents purchase and sell foreign bonds and run a trade deficit or surplus.

The paper is organized as follows. Section 2 describes the benchmark model. Section 3 presents the benchmark calibration and section 4 explains the underlying mechanisms of the dynamics of the terms of trade. Impulse response analysis is provided in section 5 as a quantitative study, and section 6 presents unconditional second moments of the model while drawing comparisons with the data. Section 7 performs a sensitivity analysis, varying several key mechanisms of the model. Section 8 concludes.

2. A Model with Endogenous Tradability of Producers

In this section, I present a two-country, dynamic, stochastic, general equilibrium (DSGE) model with endogenous non-tradability and firm-specific productivity as in Bergin & Glick (2007) and Ghironi & Melitz (2005). I augment the model with physical capital accumulation and endogenous labor choices. The world economy consists of two countries of equal size, home and foreign. Each country is populated by infinitely lived, representative consumers, perfectly competitive final goods producers, and monopolistically competitive intermediate goods producers. I assume that international financial markets are incomplete, allowing only for trade in uncontingent home and foreign bonds. There exists some risk sharing because of market friction. Unless otherwise necessary, I restrict attention to domestic agents.

2.1 The Household's Behavior

The representative household of each country chooses a combination of consumption, investment in capital and bonds, and employment to maximize lifetime utility subject to the law of motion for capital and its budget constraint. As in Backus et al. (1994, 1992), the utility function is characterized by:

$$E_0 \sum_{t=0}^{\infty} \beta^t \left[\frac{\{C_t^{\eta}(1-L_t)^{1-\eta}\}^{1-\psi}}{1-\psi} \right]$$

where C_t denotes consumption, and L_t represents hours worked. The parameter β is the subjective discount factor, η is the consumption weight in utility, and ψ is the coefficient of relative risk aversion. A unit mass of households in the home country face the following sequence of budget constraints

$$P_tC_t + P_tX_t + P_tB_{H,t+1} + P_t^*B_{F,t+1} + \frac{\xi}{2}P_tB_{H,t+1}^2 + \frac{\xi}{2}P_t^{*2}B_{F,t+1}^2 + \tilde{v}_t(N_{D,t}+N_{E,t})q_{t+1} = W_tL_t + R_tK_t + (\tilde{d}_t + \tilde{v}_t)N_{D,t}q_t + (1+i_t)P_tB_{H,t} + (1+i_t^*)P_t^*B_{F,t} + \pi_t \quad (1)$$

where P_t denotes the welfare-based price index. $B_{H,t}$ and $B_{F,t}$ are home and foreign bond holdings which pays an interest rate of i_t and i_t^* respectively. W_t is the wage rate, R_t is the rental rate of capital received from firms, and K_t is the capital stock that evolves according to $K_{t+1} = X_t + (1-\delta_k)K_t$. Here, δ_k is the capital depreciation rate. As in Boileau & Normandin (2008) and Ghironi & Melitz (2005), I assume a small quadratic portfolio cost (QPC) to avoid indeterminacy and non-stationarity. The parameter that determines the cost of adjusting the holdings of bonds, ξ, is to be small but positive. q_t is the shares in a mutual fund owned by home firms that pays an average total profit of firms $\tilde{d}_t$ as dividends.

The price of traded shares in the stock market is $\tilde{v}_t$ and therefore, $\tilde{v}_tN_{D,t}q_{t+1} + \tilde{v}_tN_{E,t}q_{t+1}$ is the total amount of resources allocated to accumulate shares in the mutual fund. $N_{D,t}$ is the number of firms that are already operating at time t, and $N_{E,t}$ is the number of new entrants. Following Ghironi & Melitz (2005), I assume there is a one period depreciation in production. Therefore, at time t+1, only $(1-\delta_d)(N_{D,t}+N_{E,t})$ firms produce. Here, δ_d is an exogenous death shock that hits firms at the end of period t. π_t is the rebate of resources using QPC to households, which is equal to $\frac{\xi}{2}(P_tB_{H,t+1}^2 + P_t^*B_{F,t+1}^2)$ in equilibrium. Similarly, foreign households face the following sequence of budget constraints.

$$P_t^*C_t^* + P_t^*X_t^* + P_tB_{H,t+1}^* + P_t^*B_{F,t+1}^* + \frac{\xi}{2}P_tB_{H,t+1}^{*2} + \frac{\xi}{2}P_t^{*2}B_{F,t+1}^{*2} + \tilde{v}_t^*(N_{D,t}^* + N_{E,t}^*)q_{t+1}^* = W_t^*L_t^* + R_t^*K_t^* + (\tilde{d}_t^* + \tilde{v}_t^*)N_{D,t}^*q_t^* + (1+i_t)P_tB_{H,t}^* + (1+i_t^*)P_t^*B_{F,t}^* + \pi_t^* \quad (2)$$

The first order conditions to the representative household's problem are as follows. The labor-leisure condition is

$$P_tC_t\frac{1-\eta}{\eta} = W_t(1-L_t). \quad (3)$$

The Euler equations for domestic and foreign bond holdings are

$$[[C_t^\eta(1-L_t)^{1-\eta}]^{1-\psi}C_t^{-1}](1+\xi B_{H,t+1}) = \beta(1+i_{t+1})E_t[[C_{t+1}^\eta(1-L_{t+1})^{1-\eta}]^{1-\psi}C_{t+1}^{-1}] \quad (4)$$

$$[[C_t^\eta(1-L_t)^{1-\eta}]^{1-\psi}C_t^{-1}](1+\xi B_{F,t+1})\frac{P_t^*}{P_t} = \beta(1+i_{t+1}^*)E_t[[C_{t+1}^\eta(1-L_{t+1})^{1-\eta}]^{1-\psi}C_{t+1}^{-1}\frac{P_{t+1}^*}{P_{t+1}}] \quad (5)$$

The Euler equation for shares in a mutual fund is

$$\tilde{v}_t = \beta(1-\delta_d)E_t\left[\left[\frac{(C_{t+1}^\eta(1-L_{t+1})^{1-\eta})^{1-\psi}C_{t+1}^{-1}}{(C_t^\eta(1-L_t)^{1-\eta})^{1-\psi}C_t^{-1}}\right](\tilde{v}_{t+1}+\tilde{d}_{t+1})\right]. \quad (6)$$

The Euler equation for the capital accumulation is

$$[C_t^\eta(1-L_t)^{1-\eta}]^{1-\psi}C_t^{-1} = \beta E_t[[C_{t+1}^\eta(1-L_{t+1})^{1-\eta}]^{1-\psi}C_{t+1}^{-1}(r_{t+1}+1-\delta_k)]. \quad (7)$$

2.2 Final Goods Producers' Behavior

Final goods in the home country are produced by aggregating a set of domestically produced varieties of intermediate goods $\Lambda_{D,t}$ and foreign produced imported varieties of intermediate goods $\Lambda^*_{X,t}$. Since there is an entry and exit of firms along the business cycle, the set of intermediate goods $\Lambda_{D,t}$ and $\Lambda^*_{X,t}$ are time varying. The maximization problem of the final goods producer is

$$\max_{f_{D,t}(a),f_{X,t}^*(a)} P_tF_t - \int_{a\in\Lambda_{D,t}} p_{D,t}(a)f_{D,t}(a)da - \int_{a\in\Lambda_{X,t}^*} p_{X,t}^*(a)f_{X,t}^*(a)da \quad (8)$$

subject to the technology that combine home and foreign produced intermediate goods as in Armington (1969):

$$F_t = \left[\left\{\int_{a\in\Lambda_{D,t}} f_{D,t}(a)^{\frac{\theta-1}{\theta}}da\right\}^{\frac{\theta}{\theta-1}\frac{\gamma-1}{\gamma}} + \left\{\int_{a\in\Lambda_{X,t}^*} f_{X,t}^*(a)^{\frac{\theta-1}{\theta}}da\right\}^{\frac{\theta}{\theta-1}\frac{\gamma-1}{\gamma}}\right]^{\frac{\gamma}{\gamma-1}}$$

where γ is the elasticity of substitution between domestic and foreign varieties of intermediate goods, and θ is elasticity of substitution within domestic varieties. Dixit & Stiglitz (1977) refer to θ a "love of variety" parameter in which a greater number of varieties is available, more goods are produced, and more consumers are satisfied. If θ is equal to γ, then the aggregation of the final goods turn out:

$$F_t = \left[\int_{a\in\Lambda_{D,t}} f_{D,t}^{\frac{\gamma-1}{\gamma}}(a)da + \int_{a\in\Lambda^*_{X,t}} f_{X,t}^{*\frac{\gamma-1}{\gamma}}(a)da\right]^{\frac{\gamma}{\gamma-1}}. \tag{9}$$

The function $f_{D,t}(a)$ is the demand for the domestic variety a and, $f_{X,t}{}^*(a)$ represents the home country importation of the foreign country's variety a. P_t denotes the price of final goods, and $p_{D,t}(a)$ and $p^*{}_{X,t}(a)$ are the prices of home and foreign variety a. The solution to this problem gives the following demand functions:

$$f_{D,t}(a) = \left[\frac{P_t F_t^{\frac{1}{\gamma}}\left(\int_{a\in\Lambda_{D,t}} f_{D,t}(a)^{\frac{\theta-1}{\theta}}da\right)^{\frac{\gamma-\theta}{\gamma(\theta-1)}}}{p_{D,t}(a)}\right]^{\theta}$$

$$f^*_{X,t}(a) = \left[\frac{P_t F_t^{\frac{1}{\gamma}}\left(\int_{a\in\Lambda^*_{X,t}} f^*_{X,t}(a)^{\frac{\theta-1}{\theta}}da\right)^{\frac{\gamma-\theta}{\gamma(\theta-1)}}}{p^*_{X.t}(a)}\right]^{\theta},$$

Aggregating yields

$$\begin{aligned}\int_{a\in\Lambda_{D,t}} f_{D,t}(a)^{\frac{\theta-1}{\theta}}da &= P_t^{\theta-1}F_t^{\frac{\theta-1}{\gamma}}\left[\int_{a\in\Lambda_{D,t}} f_{D,t}(a)^{\frac{\theta-1}{\theta}}da\right]^{\frac{\gamma-\theta}{\gamma}}\int p_{D,t}(a)^{1-\theta}da \\ &= P_t^{\frac{\gamma(\theta-1)}{\theta}}F_t^{\frac{\theta-1}{\theta}}\left[\int_{a\in\Lambda_{D,t}} p_{D,t}(a)^{1-\theta}da\right]^{\frac{\gamma}{\theta}}.\end{aligned}$$

Similarly, I can derive an expression for $\int_{a\in\Lambda^*_{X,t}} f^*_{X,t}(a)^{\frac{\theta-1}{\theta}}da.$ Substituting these expressions into the first-order conditions gives the input demand functions:

$$f_{D,t}(a) = \left(\frac{p_{D,t}(a)}{P_t}\right)^{-\theta}\left(\frac{P_{D,t}}{P_t}\right)^{\theta-\gamma}F_t \tag{10}$$

$$f^*_{X,t}(a) = \left(\frac{p^*_{X,t}(a)}{P_t}\right)^{-\theta}\left(\frac{P^*_{X,t}}{P_t}\right)^{\theta-\gamma}F_t, \tag{11}$$

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where $P_{D,t} \equiv \left[\int_{a\in\Lambda_{D,t}} N_{D,t} p_{D,t}^{1-\theta}(a)da\right]^{\frac{1}{1-\theta}}$ and $P_{X,t}^{*} \equiv \left[\int_{a\in\Lambda_{X,t}^{*}} N_{X,t}^{*} p_{X,t}^{*1-\theta}(a)da\right]^{\frac{1}{1-\theta}}$. The zero-profit condition implies that the price of final goods are

$$P_t = \left[P_{D,t}^{1-\gamma} + P_{x,t}^{*1-\gamma}\right]^{\frac{1}{1-\gamma}}. \tag{12}$$

2.3 Intermediate Goods Producers' Behavior

2.3.1 Export Decision - domestic sales or export sales?

There is a number of firms $N_{D,t}$ producing and selling in domestic markets. These firms maximize profits while taking the demand functions for their variety as given. Individual firms are differentiated by their productivity and engage in exporting activity. Firms that have export sales pay an ice-berg type trade cost τ_t per unit of intermediate goods. In addition, there exists a per period fixed export cost $f_{x,t}$, so that only an endogenous subset of firms find exporting profitable. The cost is measured in effective units of labor. Whether firms are involved in exporting activity depends on entry. Given these definitions, monopolistically competitive, intermediate goods producers maximize (13) subject to (14), (15), and (16).

$$\max p_{D,t}(a)f_{D,t}(a) + \phi(t)p_{X,t}(a)f_{X,t}(a) - mc_t(a)(f_{D,t}(a) + \phi(t)\tau_t f_{X,t}(a)) - \phi(t)\frac{W_t}{Z_t}f_{x,t} \tag{13}$$

$$f_{D,t}(a) + \varphi(t)\tau_t f_{X,t}(a) = Z_t a K_t^{\alpha}(a) L^{1}{}_{t}^{-\alpha}(a) = F_t(a) \tag{14}$$

$$f_{D,t}(a) = \left(\frac{p_{D,t}(a)}{P_t}\right)^{-} \left(\frac{P_{D,t}}{P_t}\right)^{-\gamma} F_t \tag{15}$$

$$f_{X,t}(a) = \left(\frac{p_{X,t}(a)}{P_t^{*}}\right)^{-\theta} \left(\frac{P_{X,t}}{P_t^{*}}\right)^{\theta-\gamma} F_t^{*} \tag{16}$$

where $\varphi(t)$ is an indicator function that gives 1 for firms which have export sales and 0 for firms which have domestic sales only. The output is produced with a constant return to scale production technology with identical factor shared across varieties, using capital and labor as inputs. The marginal cost of production is $mc_t(a) = \left(\frac{1}{1-\alpha}\right)^{1-\alpha}\left(\frac{1}{\alpha}\right)^{\alpha}\frac{1}{aZ_t}W_t^{1-\alpha}R_t^{\alpha}$.

Here, the parameter α is the share of income that goes to capital. All firms are subject to a common aggregate productivity factor Z_t that evolves stochastically in the model. Z_t affects the production of all goods homogeneously, while a is the firm-specific productivity. The optimization problem yields:

$$p_{D,t}(a) = \frac{\theta}{\theta-1}mc_t(a) \tag{17}$$

$$p_{X,t}(a) = \frac{\theta}{\theta-1}mc_t(a)\tau_t \tag{18}$$

Total profits for firms in t are $d_t(a) = d_{D,t}(a) + d_{X,t}(a)$. Profits for firms serving domestic sales $d_{D,t}(a)$ is obtained by substituting optimal factors of production and input demand functions back into profits:

$$\begin{aligned} d_{D,t}(a) &= p_{D,t}(a) f_{D,t}(a) - mc_t(a) f_{D,t}(a) \\ &= \frac{1}{\theta} p_{D,t}^{1-\theta}(a) P_t^{\gamma} P_{D,t}^{\theta-\gamma} F_t. \end{aligned} \tag{19}$$

Profits for firms serving export sales depends on the firms' export status. A firm exports only if it earns positive profits from exporting. Therefore, if a firm exports ($\varphi(t) = 1$), its profits are:

$$\begin{aligned} d_{X,t}(a) &= p_{x,t}(a) f_{X,t}(a) - mc_t(a) \tau_t f_{X,t}(a) - f_{x,t} \frac{W_t}{Z_t} \\ &= \frac{1}{\theta} p_{X,t}^{1-\theta}(a) P_t^{*\gamma} P_{X,t}^{\theta-\gamma} F_t^* - f_{x,t} \frac{W_t}{Z_t}. \end{aligned} \tag{20}$$

As in Ghironi & Melitz (2005) and Melitz (2003), positive profits occur when productivity a is above a cutoff productivity $a_{X,t} = inf\{a : d_{X,t}(a) > 0\}$. I assume that the minimum productivity a_{min} is low enough compare to the export costs so that exporting productivity is higher than the minimum productivity. This assumption guarantees that there are always endogenously determined subset of firms that decide not to export.

2.3.2 Factors Demand Functions - Labor and Capital

The relation between capital and labor inputs is determined by $R_t K_t(a) = \frac{\alpha}{1-\alpha} W_t L_t(a)$ from the intermediate goods producers' problem(note 3) Using the firm's resource constraint (14), labor can be written as follows:

$$L_t^{1-\alpha}(a) = \frac{F_t(a)}{Z_t a K_t^{\alpha}(a)} = \frac{f_{D,t}(a) + \phi(t)\tau_t f_{X,t}(a)}{Z_t a K_t^{\alpha}(a)}$$

Substituting the input demand functions, optimal pricing with the marginal cost of production function, and the relation between capital and labor in this equation gives

$$\begin{aligned} L_t(a) = &\left[\left(\frac{\theta}{\theta-1}\right)\left(\frac{1}{1-\alpha}\right)^{1-\alpha}\left(\frac{1}{\alpha}\right)^{\alpha} W_t^{1-\alpha} R_t^{\alpha}\right]^{-\theta}\left[\frac{W_t\alpha}{R_t(1-\alpha)}\right]^{-\alpha}(Z_t a)^{\theta-1} P_t^{\gamma} P_{D,t}^{\theta-\gamma} F_t \\ &+ \left[\left(\frac{\theta}{\theta-1}\right)\left(\frac{1}{1-\alpha}\right)^{1-\alpha}\left(\frac{1}{\alpha}\right)^{\alpha} W_t^{1-\alpha} R_t^{\alpha}\right]^{-\theta}\left[\frac{W_t\alpha}{R_t(1-\alpha)}\right]^{-\alpha}(Z_t a)^{\theta-1} \tau_t^{1-\theta} P_t^{*\gamma} P_{X,t}^{\theta-\gamma} F_t^* \end{aligned} \tag{21}$$

Using the relation between capital and labor $K_t(a) = L_t(a)\frac{\alpha}{1-\alpha}\frac{W_t}{R_t}$ and equation (21), the capital demand function is

$$K_t(a) = \left[\left(\frac{\theta}{\theta-1}\right)\left(\frac{1}{1-\alpha}\right)^{1-\alpha}\left(\frac{1}{\alpha}\right)^{\alpha} W_t^{1-\alpha}R_t^{\alpha}\right]^{-\theta}\left[\frac{W_t\alpha}{R_t(1-\alpha)}\right]^{1-\alpha}(Z_t a)^{\theta-1}P_t^{\gamma}P_{D,t}^{\theta-\gamma}F_t$$
$$+\left[\left(\frac{\theta}{\theta-1}\right)\left(\frac{1}{1-\alpha}\right)^{1-\alpha}\left(\frac{1}{\alpha}\right)^{\alpha} W_t^{1-\alpha}R_t^{\alpha}\right]^{-\theta}\left[\frac{W_t\alpha}{R_t(1-\alpha)}\right]^{1-\alpha}(Z_t a)^{\theta-1}\tau_t^{1-\theta}P_t^{*\gamma}P_{X,t}^{\theta-\gamma}F_t^{*} \tag{22}$$

Later, these demand functions will be aggregated.

2.3.3 Firm Entry and Exit, Number of the Firms

The process of firm entry and exit follows Melitz (2003). Potential entrants pay an entry cost f_E to draw productivity level a from a pareto distribution $\Phi(a)$ with the support of $[a_{min},\infty]$. The sunk entry cost is expressed in terms of effective units of labor. After productivity is drawn, all firms produce until they are hit by a exogenous death shock δ_d. As in Ghironi & Melitz (2005), I assume a time lag such that firms entering at time t start producing at time t+1. When prospective entrants make their entry decisions, they consider the average value $\tilde{v}_t$ of a firm. The present discounted value of the stream of expected average profits after period t+1 with exogenous probability of exit δ_d is

$$\tilde{v}_t = E_t\sum_{s=t+1}^{\infty}(\beta(1-\delta_d))^{s-t}\left[\left[\frac{(C_{t+1}^{\eta}(1-L_{t+1})^{1-\eta})^{1-\psi}C_{t+1}^{-1}}{(C_t^{\eta}(1-L_t)^{1-\eta})^{1-\psi}C_t^{-1}}\right]\tilde{d}_s\right],$$

and the free entry condition can be written by setting the discounted value of future expected average profits after t+1 equal to the sunk entry cost:

$$\tilde{v}_t = f_{E,t}\frac{W_t}{Z_t}. \tag{23}$$

$N_{H,t}$ is the total number of firms that exist at period t in the home economy. This consists of the firms that already exist, $N_{D,t}$, and the number of new entrants $N_{E,t}$. There is a one period lag in production in which a firm that enters in period t starts to produce at period t+1. Then, the evolution of the number of firms that survive to produce in period t+1 is

$$N_{D,t+1} = (1-\delta_d)(N_{D.t}+N_{E,t}) = (1-\delta_d)N_{H,t}. \tag{24}$$

2.3.4 Aggregation with Firm Averages

Similar to Melitz (2003), the average productivities for all producing firms $\tilde{a}_D$ and for all home exporters $\tilde{a}_{x,t}$ are

$$\tilde{a}_D \equiv \left[\int_{a_{min}}^{\infty} a^{\theta-1}d\Phi(a)\right]^{\frac{1}{\theta-1}} \tag{25}$$

$$\tilde{a}_{x,t} \equiv \left[\frac{1}{1-\Phi(a_{x,t})}\int_{a_{x,t}}^{\infty} a^{\theta-1} d\Phi(a)\right]^{\frac{1}{\theta-1}} . \tag{26}$$

The average prices of domestic sale and export sale can be written with average productivities. They are $\tilde{p}_{D,t} = \left(\frac{\theta}{\theta-1}\right)\left(\frac{1}{1-\alpha}\right)^{1-\alpha}\left(\frac{1}{\alpha}\right)^{\alpha}\frac{W_t^{1-\alpha}R_t^{\alpha}}{\tilde{a}_D Z_t}$ and $\tilde{p}_{X,t} = \left(\frac{\theta}{\theta-1}\right)\left(\frac{1}{1-\alpha}\right)^{1-\alpha}\left(\frac{1}{\alpha}\right)^{\alpha}\frac{W_t^{1-\alpha}R_t^{\alpha}}{\tilde{a}_{x,t} Z_t}$.

Previously, $P_{D,t}$ and $P_{X,t}$ are defined as $\left[\int_{a\in\Lambda_{D,t}} N_{D,t} p_{D,t}^{1-\theta}(a) da\right]^{\frac{1}{1-\theta}}$ and $\left[\int_{a\in\Lambda_{X,t}} N_{X,t} p_{X,t}^{1-\theta}(a) da\right]^{\frac{1}{1-\theta}}$ respectively; now, they can be written as $P_{D,t} = N_{D,t}^{\frac{1}{1-\theta}}\tilde{p}_{D,t}$ and $P_{X,t} = N_{X,t}^{\frac{1}{1-\theta}}\tilde{p}_{X,t}$. Plugging these equations into the price index (12) gives

$$P_t = \left[N_{D,t}^{\frac{1-\gamma}{1-\theta}}\tilde{p}_{D,t}^{1-\gamma} + N_{X,t}^{*\frac{1-\gamma}{1-\theta}}\tilde{p}_{X,t}^{*1-\gamma}\right]^{\frac{1}{1-\gamma}} . \tag{27}$$

Continuing with the results above, the average total profits are $\tilde{d}_t = \tilde{d}_{D,t} + (1-\Phi(a_{x,t}))\tilde{d}_{X,t}$. Using average prices, the average profits of domestic sales $\tilde{d}_{D,t}$ and exporting sales $\tilde{d}_{X,t}$ are

$$\tilde{d}_{D,t} = \frac{1}{\theta} N_{D,t}^{\frac{\theta-\gamma}{1-\theta}}\tilde{p}_{D,t}^{1-\gamma} P_t^{\gamma} F_t , \tag{28}$$

$$\tilde{d}_{X,t} = \frac{1}{\theta} N_{X,t}^{\frac{\theta-\gamma}{1-\theta}}\tilde{p}_{X,t}^{1-\gamma} P_t^{*\gamma} F_t^{*} - f_{x,t}\frac{W_t}{Z_t} . \tag{29}$$

Finally, aggregating the factor demand functions for labor (21) and capital (22) gives

$$L_t = \left[\left(\frac{\theta}{\theta-1}\right)\left(\frac{1}{1-\alpha}\right)^{1-\alpha}\left(\frac{1}{\alpha}\right)^{\alpha} W_t^{1-\alpha}R_t^{\alpha}\right]^{-\theta}\left[\frac{W_t\alpha}{R_t(1-\alpha)}\right]^{-\alpha} Z_t^{\theta-1} \left[\tilde{a}_D^{\theta-1}P_t^{\gamma}\left(N_{D,t}^{\frac{1}{1-\theta}}\tilde{p}_{D,t}\right)^{\theta-\gamma}F_t + \tilde{a}_{x,t}^{\theta-1}\tau_t^{1-\theta}P_t^{*\gamma}\left(N_{X,t}^{\frac{1}{1-\theta}}\tilde{p}_{X,t}\right)^{\theta-\gamma}F_t^{*}\right]$$

$$K_t = \left[\left(\frac{\theta}{\theta-1}\right)\left(\frac{1}{1-\alpha}\right)^{1-\alpha}\left(\frac{1}{\alpha}\right)^{\alpha} W_t^{1-\alpha}R_t^{\alpha}\right]^{-\theta}\left[\frac{W_t\alpha}{R_t(1-\alpha)}\right]^{1-\alpha} Z_t^{\theta-1} \left[\tilde{a}_D^{\theta-1}P_t^{\gamma}\left(N_{D,t}^{\frac{1}{1-\theta}}\tilde{p}_{D,t}\right)^{\theta-\gamma}F_t + \tilde{a}_{x,t}^{\theta-1}\tau_t^{1-\theta}P_t^{*\gamma}\left(N_{X,t}^{\frac{1}{1-\theta}}\tilde{p}_{X,t}\right)^{\theta-\gamma}F_t^{*}\right]$$

Research in Applied Economics
ISSN 1948-5433
2019, Vol. 11, No. 3

2.4 Market Clearing Conditions and Equilibrium

The equilibrium for this economy requires several market-clearing conditions. The model is closed by the bond market clearing condition $B_{H,t+1}+B^*_{H,t+1}=0$ and $B^*_{F,t+1}+B_{F,t+1}=0$ and the value of shares in the mutual fund market clearing condition $q_{t+1}=q^*_{t+1}=1$. Subtracting the foreign budget constraints (2) from the home budget constraints (1) and then applying bond and the mutual fund market clearing conditions gives the balance of payment condition as follows.

$$\begin{aligned} P_tB_{H,t+1}+P^*_tB_{F,t+1} &= P_t(1+i_t)B_{H,t}+P^*_t(1+i^*_t)B_{F,t}+\frac{1}{2}\left(W_tL_t-W^*_tL^*_t\right) \\ &+\frac{1}{2}\left(R_tK_t-R^*_tK^*_t\right)-\frac{1}{2}\left(P_tC_t-P^*_tC^*_t\right)-\frac{1}{2}\left(P_tX_t-P^*_tX^*_t\right) \\ &+\frac{1}{2}\left(N_{D,t}\tilde{d}_t-N^*_{D,t}\tilde{d}^*_t\right)-\frac{1}{2}\left(N_{E,t}\tilde{v}_t-N^*_{E,t}\tilde{v}^*_t\right) \end{aligned} \tag{30}$$

Given the laws of motion for capital and aggregate shocks, an equilibrium for this economy is a set of allocations for home and foreign consumers, home and foreign final goods producers, home and foreign intermediate goods producers, their prices, and market clearing conditions such that: the labor-leisure condition, Euler Equations to bonds, shares, and capital stock, free entry condition, number of firms, price indexes, profits, zero-profit export cutoffs, share of exporting firms, factor demand functions for labor and capital stock in production, final goods resource constraint, and balance of payment condition. The benchmark model economy and its associated steady state system have 45 independent equations and 45 variables which must be solved for: 23 home variables ($P_t, C_t, L_{,t}, K_t, X_t, B_{H,t}, B_{F,t}, R_t, i_t, W_t, F_t, \tilde{a}_{x,t}, \tilde{v}_t, \tilde{d}_t, \tilde{d}_{D,t}, \tilde{d}_{X,t}, NP_{X,t}$) and 22 foreign variables ($P^*_t, C^*_t, L^*_t, K^*_t, X^*_t, B^*_{F,t}, B^*_{H,t}, R^*_t, i^*_t, F^*_t, \tilde{a}^*_{x,t}, \tilde{v}^*_t, \tilde{d}^*_t, \tilde{d}^*_{D,t}, \tilde{d}^*_{X,t}, N^*_{E,t}$, $N^*_{D,t}, N^*_{X,t}, \tilde{p}^*_{D,t}, \tilde{p}^*_{X,t}$). The foreign wage W^*_t is set equal to 1. The full summary of 45 system of equations is provided in the Appendix.

2.5 Aggregate Productivity Shocks

In the benchmark model, shocks to aggregate productivities are introduced. For this, I use a bivariate, autoregressive process for percent deviations of home and foreign aggregate productivity from their steady state. The symmetric and exogenous process for total factor productivity can be written in the following (log-linearized form):

$$\begin{bmatrix}\tilde{z}_t\\ \tilde{z}^*_t\end{bmatrix}=\begin{bmatrix}\rho & \rho_{HF}\\ \rho_{FH} & \rho^*\end{bmatrix}\begin{bmatrix}\tilde{z}_{t-1}\\ \tilde{z}^*_{t-1}\end{bmatrix}+\begin{bmatrix}\varepsilon_t\\ \varepsilon^*_t\end{bmatrix} with \begin{bmatrix}\varepsilon_t\\ \varepsilon^*_t\end{bmatrix}\sim N\left(\begin{bmatrix}0\\0\end{bmatrix},\begin{bmatrix}\sigma^2_\varepsilon & \sigma_{\varepsilon\varepsilon^*}\\ \sigma_{\varepsilon\varepsilon^*} & \sigma^2_{\varepsilon_*}\end{bmatrix}\right)$$

3. Benchmark Calibration

I chose benchmark values for the set of relevant parameters so as to match features of the US economy at a quarterly frequency. In accordance with the literature, the intertemporal discount factor of households β is 0.99. The risk aversion coefficient ψ is 2, and the consumption weight in households' utility η is 0.378, so that 1/3 of the time endowment is devoted to work in the steady state. The rate of physical capital depreciation δ_k is 0.025 and the capital share α is 0.36, so that the capital income share is close to 30%. The quadratic adjustment cost of bond holdings is set to $\xi = \beta^2 * 0.01$ as in Boileau & Normandin (2008). The value of the ice-berg transport cost is set to 1.7 as in Alessandria & Choi (2007). Closely following the work of Bergin & Glick (2007) and Bernard, Eaton, Jensen & Kortum (2003), the fixed exporting costs parameter f_x is set to 0.0065 to ensure that the share of exporting plants is 21% in the steady state. As in Ghironi & Melitz (2005), I assume that the sunk entry costs parameter f_E is 1. The elasticity of substitution between intermediate goods and within domestic varieties are set to 3.82 and 3.8 respectively, so that they deliver a 20% markup of price over marginal cost. The values for trade elasticity of substitution is controversial. Therefore, I vary them and study how parameterizations of trade elasticity effect the numerical results. The minimum value of firm specific productivity a_{min} is set equal to 1. Here, I also adhere closely to Ghironi & Melitz (2005) for the shape parameter κ of Pareto distribution $\Phi(a) = 1 - \left(\frac{a_{min}}{a}\right)^{\kappa}$, which is set equal to 3.4.

Table 1. Benchmark Parameter Values

Description	value
Consumption weight in utility	η=0.378 (L=1/3)
Coefficient of relative risk aversion	ψ=2
Capital income share	α=0.36
Intertemporal discount factor	β=0.99
Capital depreciation rate	δ_k=0.025
Probability of death shock	δ_d=0.024
Quadratic adjustment costs of bond holdings	ξ=β^2*0.01
Ice-berg transport cost	τ=1.7
Fixed exporting costs parameter	f_x=0.0065
Sunk entry costs parameter	f_E=1
Elasticity of substitution between intermediate goods	γ=3.5
Elasticity of substitution within domestic varieties	θ=3.5
Lower bound of productivity	a_{min}=1
Exporting productivity parameter	a_x=2.8
Characterizing parameter of $\Phi(a)$	κ=3.4
Number of exporters out of number of producing firms	$\frac{N_{X,t}}{N_{D,t}} = 0.34^{\kappa}$
Persistence parameter of aggregate productivity shock	ρ=0.906
Spillover parameter of aggregate productivity shock	ρ_{HF}=0.088

As in Backus et al. (1994), the persistence of the aggregate productivity shock (ρ, ρ^*) is set to 0.906. The spill over parameter ρ_{HF} , ρ_{FH} are set to 0.088. The standard deviation of productivity innovations is 0.00852 and the correlation between productivity innovations is 0.258. Table 3 lists all calibrated parameters.

4. International Relative Prices Dynamics

The international relative price is defined as the price of imports over the price of producers (the opposite of Bergin & Glick (2007) and Obstfeld & Rogoff (2000)). The standard international real business cycle model fails to account for the appreciation of the terms of trade in response to a positive total factor productivity shock. Backus et al. (1994) explain that the relative price of domestically produced goods decreases with productivity shocks, which implies depreciation in the terms of trade. The novel finding of this paper is that the benchmark model can account for the observed dynamics of the terms of trade by arguing that endogenous tradability along with heterogenous firms and endogenous capital and labor cause the terms of trade to appreciate.

From the model, the terms of trade can be written as

$$\begin{aligned} TOT_t = \frac{P_{IMP}}{P_{EXP}} &= \frac{{N^*_{X,t}}^{\frac{1}{1-\theta}} \tilde{p}^*_{X,t}}{N_{X,t}^{\frac{1}{1-\theta}} \tilde{p}_{X,t}} \\ &= \left(\frac{N^*_{X,t}}{N_{X,t}}\right)^{\frac{1}{1-\theta}} \left(\frac{\tilde{a}_{x,t}}{\tilde{a}^*_{x,t}}\right) \left(\frac{\frac{W_t^{*1-\alpha} R_t^{*\alpha} \tau_t^*}{Z_t^*}}{\frac{W_t^{1-\alpha} R_t^{\alpha} \tau_t}{Z_t}}\right). \end{aligned} \tag{31}$$

This equation can be decomposed into three parts: the relative number of exporters, the relative cut-off exporting productivity, and the relative costs of units of factors across countries, such as labor and capital. In log-linearized form, the terms of trade are

$$\begin{aligned} \hat{TOT}_t = &\left(\frac{1}{1-\theta}\right) \hat{N}^*_{X,t} + \hat{Z}_t + \hat{\tilde{a}}_{x,t} + (1-\alpha)\hat{w}^*_t + \alpha \hat{r}^*_t \\ &- \left(\frac{1}{1-\theta}\right) \hat{N}_{X,t} - \hat{Z}^*_t - \hat{\tilde{a}}^*_{x,t} - (1-\alpha)\hat{w}_t - \alpha \hat{r}_t. \end{aligned} \tag{32}$$

The hatted variables denote percentage deviations from the steady state. The theoretical mechanisms behind of the appreciation of the terms of trade are explained below in more detail. There are two effects that move terms of trade in different directions. One is the GDP effect which causes the terms of trade to increase (depreciate) in response to a positive productivity shock in the home economy; the other is the factor efficiency effect that causes the terms of trade to decrease (appreciate). GDP in the home economy increases when there is a 1% positive productivity shock. Increase in the home income stimulate the demands for varieties. It is understood that even the less productive foreign firms in the model can enter the home market

Research in Applied Economics
ISSN 1948-5433
2019, Vol. 11, No. 3

to meet the increased demand. Therefore, the exporting cutoff productivity for foreign firms decreases and then, the second component of the terms of trade, the relative cut-off exporting productivity increases. This GDP effect, in turn, causes the terms of trade to depreciate. However, there is the factor efficiency effect that moves against the GDP effect. When there is domestic productivity shock, factor production costs decrease and bring more firms to enter the home market. The final component of the terms of trade manifests itself when the terms of efficiency decrease due to decrease in production costs in home. More entry into the home market generates competition among firms and this prohibits less productive foreign firms from entering the home market. This means that the foreign cut-off productivity $\tilde{a}^{*}_{x,t}$ increases in response to an aggregate productivity shock while the relative cut-off productivity decreases as do the terms of trade. In summary, there are both GDP and factor efficiency effects, but since the efficiency effect is larger than the GDP effect, this causes the terms of trade to appreciate. Figure 1 displays the dynamic response of the key variables for the terms of trade dynamics to 1% increase in home productivity. It matches the theoretical mechanism described above.

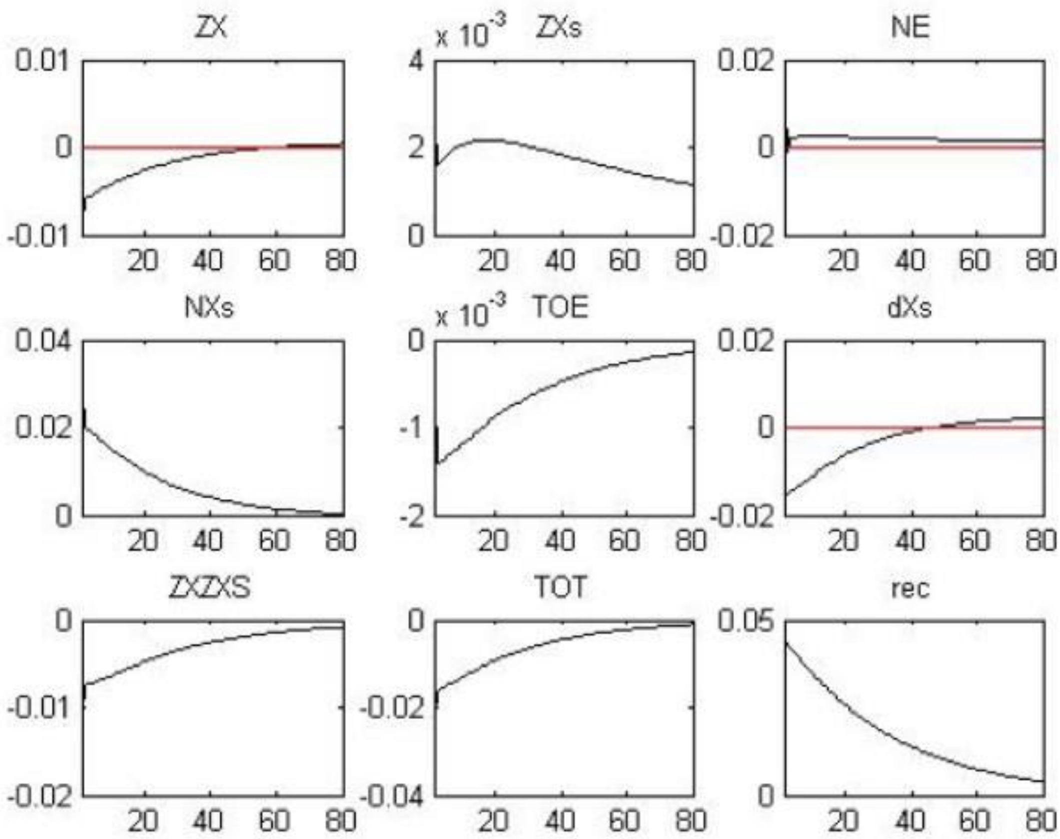

Figure 1. Terms of Trade Dynamics to 1% Increase in Home Productivity

Note: These variables are the exporting cut-off productivity of home firms (ZX) and foreign firms (ZXs), the new entrants (NE), the number of foreign exporters (NXs), the terms of efficiency (TOE), the foreign exporting profits (dXs), the relative exporting cut-off productivity (ZXZXS), the terms of trade (TOT), and relative consumption (rec).

Research in Applied Economics
ISSN 1948-5433
2019, Vol. 11, No. 3

5. Impulse Response Analysis

In this section, I explore the quantitative properties of the benchmark model through impulse responses. I solve the baseline model, log-linearizing the 45 systems of equations around the steady state and solving the resulting system of linear difference equations as described in King, Plosser & Rebelo (2002), and apply Uhlig (1995) techniques. Given the parameters that characterize behavior around the steady states and the law of motion of shocks, DYNARE and MATLAB program were used for simulation. The impulse response results to aggregate productivity shocks in the home economy are presented here so that the mechanisms of the model can be explained and further intuitions can be gained.

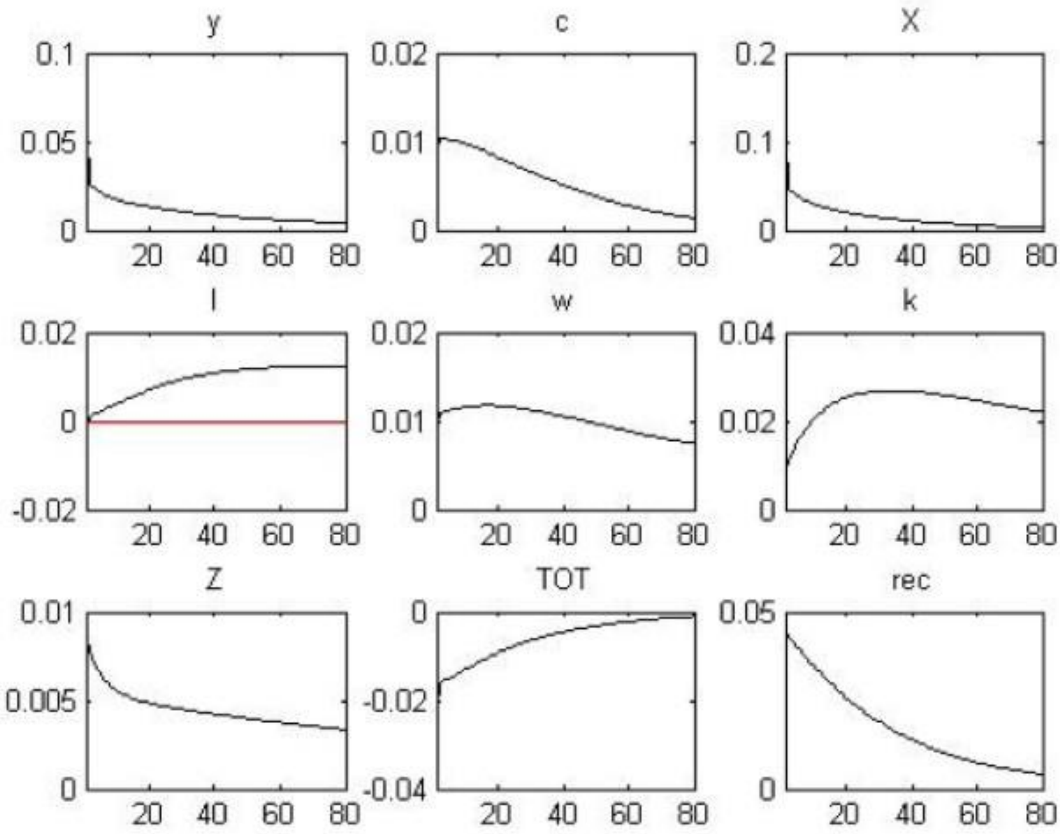

Figure 2. Response to 1% Increase in Home Productivity

Note: These variables are home country's GDP (y), consumption (C), investment (X), employment (l), wage rate (w), capital (k), productivity (Z), terms of trade (TOT), and relative consumption (rec).

Figure 2 displays the dynamic response of key macroeconomic variables to a 1% increase in home productivity. This aggregate productivity shock is followed by an initial increase in GDP before falling sharply. I expect that the temporary nature of the increase comes from the one period time lag built into the model, which constrains entrants who enter at time t from starting production until time t+1. Consumption, investment, and employment rises with a home aggregate productivity shock as resources are transmitted to more productive locations. The magnitude of the increase in GDP is larger than that of consumption. It means that the home country accumulates the net foreign assets. The home country can produce labor and capital more cheaply; therefore, input costs decrease, but factor efficiency increases as productivity rises. In other words, the international relative efficiency decreases. A positive productivity shock results in more entry into the home market, so the number of foreign exporters increases and generates more competition among firms. The subsequent increase in competition means that only the more productive foreign firms can enter the home market. Therefore, the exporting cut-off productivity for foreign producers increases in response to a productivity shock. Home consumption and foreign consumption both increase with a home productivity shock, but foreign consumption and relative consumption increases less than in the home market. Therefore, I obtain the negative correlation between relative consumption across countries and the terms of trade.

6. International Real Business Cycle Moments

In this section, unconditional second moments are presented using a benchmark model and later compared to the facts commonly observed in real world economic data. I use the model to confront several observations on the business cycle statistics. The Hodrick and Prescott (HP)(note 4) filter is applied to compute the model's statistics by logging and filtering the model's artificial time series. Table 2 summarizes the main statistics of the simulated model under the benchmark parameters at business cycle frequencies. I compare this to the work Ghironi & Melitz (2005), which does not incorporate capital accumulation and endogenous labor, and to statistics of the bond economy with no capital adjustment costs taken from Kehoe & Perri (2002). The data for the correlation between relative consumption and the terms of trade is taken from Corsetti et al. (2008).

I study the model's implications for the (1) standard deviations of a variable relative to that of the logarithm of output, (2) correlation between a variable and the logarithm of output as a domestic co-movement, (3) international correlations between home variables and foreign variables, and the (4) correlation between relative consumption and the terms of trade as an other correlation. I compare the benchmark model with the data and the simulated model of Ghironi & Melitz (2005) and Kehoe & Perri (2002). In Ghironi & Melitz (2005) model, capital does not exist and labor supply is fixed. A similar pattern of aggregate volatilities are observed in the benchmark model and in the data. Investment is most volatile variable among the six key variables, while consumption and employment are less volatile than GDP. In addition, the benchmark model also correctly predicts that key variables are procyclical with GDP. Entry is the most volatile in their model, while the volatility of entry is dampened in the benchmark model with endogenous capital and labor. Imposing asset market frictions lets home agents purchase and sell foreign bonds, which causes either a trade deficit or surplus. Therefore, asset market frictions help to explain the reduction in the volatility of entry because each country tries to maintain balanced trade.

It is common for researchers to include capital adjustment costs so that the volatility of investment can be reduced with the higher productivity shocks. The benchmark model and

Kehoe & Perri (2002)'s model do not have this capital adjustment cost feature; however, the benchmark model has much less volatility of investment. This implies that incorporating entry and exporting decisions in aggregate business cycle fluctuations dampens the variability of investment. In the benchmark model, extensive margin works similarly to capital adjustment cost. The entry condition is determined in terms of effective labor, so it uses labor more than capital goods. Therefore, even without the capital adjustment costs, the volatility of investment is reduced.

Standard IRBC model produce higher cross-country consumption correlations than output correlations, but this is the opposite of what is present in the data (Backus et al. (1992) calls this "quantity anomaly"). The simulation result shows that augmenting capital accumulation in the entry process does not help solving the quantity anomaly. The international correlation of output is -0.87 and the consumption is 0.41. In addition, the benchmark model shares the failure of international co-movement of GDP, investment, and labor. The correlations between the international consumption ratio and the terms of trade is -0.86, so the benchmark model solves the Backus-Smith puzzle! The Backus-Smith puzzle is an anomaly in which standard IRBC model predicts that the terms of trade is positively correlated to the relative consumption across countries, although the data shows they are negatively correlated or close to zero.

Table 2. Business Cycle Statistics: Baseline Parameters

	Data	Benchmark	GM(05)	KP(02)
Volatility				
% S.D. relative to GDP				
GDP	1	1	1	1
Consumption (C)	0.72	0.52	0.59	0.21
Investment (X)	3.87	2.99	_	25.06
Employment (L)	0.58	0.58	_	0.54
Entry (N_E)	_	1.69	4.57	_
Terms of Trade (TOT)	1.44	0.32	0.035	_
Domestic Comovement				
Correlations with GDP				
Consumption	0.86	0.70	1	0.93
Investment	0.89	0.91	_	0.08
Employment	0.79	0.61	_	0.99
International Correlations				
GDP, GDP^*	0.55	-0.87	0.21	-0.43
C, C^*	0.42	0.21	0.86	0.13
X, X^*	0.39	-0.89	_	-0.99
L, L^*	0.28	-0.23	_	-0.53
N_E, N_E^*	_	-0.04	_	_
Other Correlation				
Consumption ratio, TOT	-0.35(CDL)	-0.93	-0.99	_

Research in Applied Economics
ISSN 1948-5433
2019, Vol. 11, No. 3

7. Robustness Analysis

To understand the robustness of the main results under different assumptions, two additional cases are considered. In the first case, the elasticity of substitution between domestic and foreign goods are allowed to vary. It is critical parameter because it determines the behavior of trade flows and international prices. In the second case, it is the aggregate productivity shock process that varies.

7.1 Elasticity of Substitution between Intermediate Goods

Table 3. Elasticity of Substitution between Intermediate Goods: γ

	Data	γ=1.2	γ=3	γ=3.5	γ=4	γ=5	γ=6
Volatility							
% S.D. relative to GDP							
GDP	1	1	1	1	1	1	1
Consumption (C)	0.72	0.38	0.48	0.52	0.64	0.86	0.98
Investment (X)	3.87	2.72	2.87	2.99	2.98	2.93	2.91
Employment (L)	0.58	0.47	0.41	0.58	0.66	0.69	0.68
Entry (N_E)	_	1.82	1.74	1.69	1.44	1.53	1.36
Terms of Trade (TOT)	1.44	0.36	0.30	0.32	0.38	0.49	0.46
International Correlations							
GDP, GDP^*	0.55	-0.55	-0.79	-0.87	-0.85	-0.89	-0.91
C, C^*	0.42	0.77	0.89	0.21	-0.14	-0.22	-0.31
X, X^*	0.39	-0.49	-0.70	-0.89	-0.91	-0.94	-0.95
L, L^*	0.28	0.79	0.87	-0.23	0.04	-0.28	-0.28
Other Correlation							
Consumption ratio, TOT	-0.35 (CDL)	-0.71	-0.87	-0.93	-0.95	-0.97	-0.95

In the benchmark model, the elasticity of substitution between intermediate goods (γ) is set to 3.5 and the elasticity of substitution within domestic varieties (θ) is set to 3.8. In this exercise, θ is fixed as 3.8 and then γ varies from 1.2 to 6. Smaller γ means that domestic and foreign goods are easily substitutable. When domestic and foreign goods are more complementary, the volatility of consumption, investment, and employment are all lower. When domestic and foreign goods are more substitutable, the reallocation effect, in which factors are distributed from the less productive country to the more efficient one, is reduced. This reduction in the reallocation effect leads to a reduced volatility of consumption, investment, and employment.

It is common for researchers to solve co-movement puzzles using low elasticity of substitution between domestic and foreign production in the IRBC literature. However, empirical trade literature sets this parameter high. The benchmark model is able to work with the high trade literature, but with the model still robust enough to solve anomalies. In my model, the international output correlations are smaller than international consumption correlations ('quantity anomaly') when γ varies. However, when γ is smaller (intermediate goods are more complementary), so this relationship has improved. The cross-correlation of output is still negative, but it increases when γ is smaller. In addition, a lower elasticity of substitution between intermediate goods reduces the consumption effect with respect to domestic goods and reduces the selection effect. The correlation between relative consumption and the terms of trade is still negative with smaller γ, but this correlation is larger when γ is lower.

To summarize, the benchmark model responds in the business cycle moments properly to these changes in the specific parameter value γ. At the same time, the main forces shaping these results are identified by varying this parameter.

7.2 Baxter & Crucini (1995) Shock Process

Table 4. Baxter & Crucini (2995) Shock Process: ρ=0.999, ρ_{HF} =0

	Data	BKK (Benchmark)	BC
Volatility			
% S.D. relative to GDP			
GDP	1	1	1
Consumption (C)	0.72	0.52	0.76
Investment (X)	3.87	2.99	3.00
Employment (L)	0.58	0.58	0.89
Entry (N_E)	_	1.69	2.56
Terms of Trade (TOT)	1.44	0.32	0.37
International Correlations			
GDP, GDP^*	0.55	-0.87	-0.88
C, C^*	0.42	0.21	-0.10
X, X^*	0.39	-0.89	-0.86
L, L^*	0.28	-0.23	0.32
Other Correlation			
Consumption ratio, TOT	-0.35 (CDL)	-0.93	-0.91

The quantitative analysis shows that the benchmark model solved the terms of trade anomaly; although, it still shares the failure to solve the quantity anomaly of standard IRBC models.

Baxter & Crucini (1995) argue that an IRBC model with incomplete markets helps to solve the quantity anomaly because of imperfect risk sharing. When there is a domestic productivity shock, the rise in home productivity generates a large income increase at home, but a small increase in income abroad. The incomplete markets models driven by the Baxter & Crucini (1995) process that occurs when productivity shocks are highly persistent and do not spill over international boundaries. Here, the persistent parameter is set equal to 0.999 and the spill over parameter is set equal to 0. As a result, the business cycle implications of the benchmark model are robust to changes in the shock process. However, the international correlation of consumption turns negative and the difference between the correlations of international output and international consumption is reduced (from 1.08 to 0.78).

8. Concluding Remarks

In this paper, I focus on the problem of the anomaly of international relative prices dynamics, building a DSGE model with endogenous tradability and heterogenous firms. I show that the model successfully addresses the dynamics of the terms of trade through the exporting cut-off productivity of an endogenous trade pattern. Augmenting the model with physical capital accumulation not only improves the definition of investment, but also provides a more realistic framework. The endogenously determined physical capital accumulation can be defined as the entry of new firms, spending on capital equipment, structures, and inventories for producers' entry and exit dynamics. The quantitative analysis of the model shows that international relative prices appreciate due to a factor efficiency effect through the relative exporting cut-off productivity for home and foreign producers. The model of the paper may be extended in a number of dimensions to further investigate the properties, such as analyzing impulse response dynamics with investment-specific technology shock and estimating the impulse responses of the macroeconomic variables to shocks.

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Notes

Note 1. Obstfeld & Rogoff (2000), and Backus et al. (1992) include trade costs in their model and show that adding trade costs as trade frictions help to solve several puzzles in international macroeconomics.

Note 2. The Harrod-Balassa-Samuelson (HBS) or Balassa-Samuelson (BS) effect is that wealthier economies have higher average prices relative to their trading partners. As a result, the terms of trade or exchange rate appreciate when there is a positive aggregate productivity shock in the home economy.

Note 3. This relationship comes from optimization result in previous section: $W_t = mc_t(a)(1-\alpha)\frac{F_t(a)}{L_t(a)}$ and $R_t = mc_t(a)\alpha\frac{F_t(a)}{K_t(a)}$

Note 4. The HP filter removes the cyclical component of a time series and is commonly used in macroeconomic data.

Research in Applied Economics
ISSN 1948-5433
2019, Vol. 11, No. 3

Appendix

Appendix A. Data Sources

Data for most countries are from the International Financial Statistics (IFS) provided by the International Monetary Fund (http://elibrary-data.inf.org/). U.S. quarterly data (1973Q12009Q4) for GDP, consumption, and investment is extracted and all variables have been logged and detrended using the Hodrick-Prescott filter (smoothing parameter of 1600). U.S. labor data is obtained from the Bureau of Labor Statistics (http://www.bls.gov/) and the OECD.StatExtracts (http://stat.oecd.org). To calculate the international correlations, U.S. data and Europe aggregates are compared. The quarterly data (1973Q1-2008Q3) for GDP, consumption, investment, and civilian employment are from IFS. European countries include Austria, Finland, France, Germany, Italy, Sweden, Switzerland, and the U.K. Investment includes gross fixed capital formation and changes in inventories. Labor input per capital is calculated as hours per worker multiplied by civilian employment and then divided by population age 16 and over. I follow the tradition of the international business cycle literature in defining the terms of trade as the relative price of imports to exports.

Appendix B. Equilibrium Conditions

B.1 Benchmark Model – Incomplete Asset Markets

I list summary of 45 equilibrium system of equations of the benchmark model. Labor-Leisure condition

$$W_t(1-L_t) = P_tC_t\left(\frac{1-\eta}{\eta}\right) \tag{B.1}$$

$$1-L_t^* = P_t^*C_t^*\left(\frac{1-\eta}{\eta}\right) \tag{B.2}$$

Euler Equations (Bonds)

$$[[C_t^{\eta}(1-L_t)^{1-\eta}]^{1-\psi}C_t^{-1}](1+\xi B_{H,t+1}) = \beta(1+i_{t+1})E_t[[C_{t+1}^{\eta}(1-L_{t+1})^{1-\eta}]^{1-\psi}C_{t+1}^{-1}] \tag{B.3}$$

$$\left[\left(C_t^{\eta}(1-L_t)^{1-\eta}\right)^{1-\psi}C_t^{-1}\frac{P_t^*}{P_t}\right](1+\xi B_{F,t+1}) = \beta(1+i_{t+1}^*)E_t\left[\left\{C_{t+1}^{\eta}(1-L_{t+1})^{1-\eta}\right\}^{1-\psi}C_{t+1}^{-1}\frac{P_{t+1}^*}{P_{t+1}}\right] \tag{B.4}$$

$$\left[\left(C_t^{*\eta}(1-L_t^*)^{1-\eta}\right)^{1-\psi}C_t^{*-1}\frac{P_t}{P_t^*}\right](1+\xi B_{H,t+1}^*) = \beta(1+i_{t+1})E_t\left[\left\{C_{t+1}^{*\eta}(1-L_{t+1}^*)^{1-\eta}\right\}^{1-\psi}C_{t+1}^{*-1}\frac{P_{t+1}}{P_{t+1}^*}\right] \tag{B.5}$$

$$[[C_t^{*\eta}(1-L_t^*)^{1-\eta}]^{1-\psi}C_t^{*-1}](1+\xi B_{F,t+1}^*) = \beta(1+i_{t+1}^*)E_t[[C_{t+1}^{*\eta}(1-L_{t+1}^*)^{1-\eta}]^{1-\psi}C_{t+1}^{*-1}] \tag{B.6}$$

Euler Equations (Shares)

$$\tilde{v}_t = \beta(1-\delta_d)E_t\left[\left[\frac{(C_{t+1}^{\eta}(1-L_{t+1})^{1-\eta})^{1-\psi}C_{t+1}^{-1}P_t}{(C_t^{\eta}(1-L_t)^{1-\eta})^{1-\psi}C_t^{-1}P_{t+1}}\right](\tilde{v}_{t+1}+\tilde{d}_{t+1})\right] \tag{B.7}$$

$$\tilde{v}_t^* = \beta(1-\delta_d)E_t\left[\left[\frac{(C_{t+1}^{*\eta}(1-L_{t+1}^*)^{1-\eta})^{1-\psi}C_{t+1}^{*-1}P_t^*}{(C_t^{*\eta}(1-L_t^*)^{1-\eta})^{1-\psi}C_t^{*-1}P_{t+1}^*}\right](\tilde{v}_{t+1}^* + \tilde{d}_{t+1}^*)\right] \quad (B.8)$$

Euler Equations (Capital stock accumulation)

$$[C_t^{\eta}(1-L_t)^{1-\eta}]^{1-\psi}C_t^{-1} = \beta E_t[[C_{t+1}^{\eta}(1-L_{t+1})^{1-\eta}]^{1-\psi}C_{t+1}^{-1}(r_{t+1}+1-\delta_k)] \quad (B.9)$$

$$[C_t^{*\eta}(1-L_t^*)^{1-\eta}]^{1-\psi}C_t^{*-1} = \beta E_t[[C_{t+1}^{*\eta}(1-L_{t+1}^*)^{1-\eta}]^{1-\psi}C_{t+1}^{*-1}(r_{t+1}^*+1-\delta_k)] \quad (B.10)$$

Investment in Capital

$$K_{t+1} = X_t + (1-\delta)K_t \quad (B.11)$$

$$K_{t+1}^* = X_t^* + (1-\delta)K_t^* \quad (B.12)$$

Free Entry Conditions

$$\tilde{v}_t = f_{E,t}\frac{W_t}{Z_t} \quad (B.13)$$

$$\tilde{v}_t^* = f_{E,t}^*\frac{1}{Z_t^*} \quad (B.14)$$

Number of Firms

$$N_{D,t+1} = (1-\delta_d)\left(N_{D.t} + N_{E,t}\right) \quad (B.15)$$

$$N_{D,t+1}^* = (1-\delta_d)\left(N_{D.t}^* + N_{E,t}^*\right) \quad (B.16)$$

Price Indexes

$$P_t = \left(P_{D,t}^{1-\gamma} + P_{X,t}^{*1-\gamma}\right)^{\frac{1}{1-\gamma}} \quad (B.17)$$

$$P_t^* = \left(P_{D,t}^{*1-\gamma} + P_{X,t}^{1-\gamma}\right)^{\frac{1}{1-\gamma}} \quad (B.18)$$

$$P_{D,t} = N_{D,t}^{\frac{1}{1-\theta}}\tilde{p}_{D,t} \quad (B.19)$$

$$P_{D,t}^* = N_{D,t}^{*\frac{1}{1-\theta}}\tilde{p}_{D,t}^* \quad (B.20)$$

$$P_{X,t} = N_{X,t}^{\frac{1}{1-\theta}}\tilde{p}_{X,t} \quad (B.21)$$

$$P_{X,t}^* = N_{X,t}^{*\frac{1}{1-\theta}}\tilde{p}_{X,t}^* \quad (B.22)$$

Total Average Profits

$$\tilde{d}_t = \tilde{d}_{D,t} + (1-\Phi(a_{x,t}))\,\tilde{d}_{X,t} \quad (B.23)$$

$$\tilde{d}_t^* = \tilde{d}_{D,t}^* + \left(1-\Phi(a_{x,t}^*)\right)\tilde{d}_{X,t}^* \quad (B.24)$$

Average Profits from Domestic Sales

$$\tilde{d}_{D,t} = \frac{1}{\theta} P_{D,t}^{1-\gamma} N_{D,t}^{-1} P_t^{\gamma} F_t \tag{B.25}$$

$$\tilde{d}^*_{D,t} = \frac{1}{\theta} P_{D,t}^{*1-\gamma} N_{D,t}^{*-1} P_t^{*\gamma} F_t^* \tag{B.26}$$

Average Profits from Foreign Sales

$$\tilde{d}_{X,t} = \frac{1}{\theta} P_{X,t}^{1-\gamma} N_{X,t}^{-1} P_t^{*\gamma} F_t^* - \frac{W_t}{Z_t} f_{x,t} \tag{B.27}$$

$$\tilde{d}^*_{X,t} = \frac{1}{\theta} P_{X,t}^{*1-\gamma} N_{X,t}^{*-1} P_t^{\gamma} F_t - \frac{1}{Z_t^*} f^*_{x,t} \tag{B.28}$$

Average Prices

$$\tilde{p}_{D,t} = \left(\frac{\theta}{\theta-1}\right)\left(\frac{1}{\alpha}\right)^{\alpha}\left(\frac{1}{1-\alpha}\right)^{1-\alpha} \frac{r_t^{\alpha} W_t^{1-\alpha}}{Z_t \tilde{a}_D} \tag{B.29}$$

$$\tilde{p}_{X,t} = \left(\frac{\theta}{\theta-1}\right)\left(\frac{1}{\alpha}\right)^{\alpha}\left(\frac{1}{1-\alpha}\right)^{1-\alpha} \tau_t \frac{r_t^{\alpha} W_t^{1-\alpha}}{Z_t \tilde{a}_{x,t}} \tag{B.30}$$

$$\tilde{p}^*_{D,t} = \left(\frac{\theta}{\theta-1}\right)\left(\frac{1}{\alpha}\right)^{\alpha}\left(\frac{1}{1-\alpha}\right)^{1-\alpha} \frac{r_t^{*\alpha}}{Z_t^* \tilde{a}_D} \tag{B.31}$$

$$\tilde{p}^*_{X,t} = \left(\frac{\theta}{\theta-1}\right)\left(\frac{1}{\alpha}\right)^{\alpha}\left(\frac{1}{1-\alpha}\right)^{1-\alpha} \tau_t^* \frac{r_t^{*\alpha}}{Z_t^* \tilde{a}^*_{x,t}} \tag{B.32}$$

Zero-Profit Export Cutoffs Conditions

$$\tilde{d}_{X,t} = \frac{W_t}{Z_t} f_{x,t} \frac{\theta-1}{\kappa-(\theta-1)} \tag{B.33}$$

$$\tilde{d}^*_{X,t} = \frac{1}{Z_t^*} f^*_{x,t} \frac{\theta-1}{\kappa-(\theta-1)} \tag{B.34}$$

Share of Exporting Firms

$$1-\Phi(a_{x,t}) = \frac{N_{X,t}}{N_{D,t}} \tag{B.35}$$

$$1-\Phi(a^*_{x,t}) = \frac{N^*_{X,t}}{N^*_{D,t}} \tag{B.36}$$

Demand Functions for Labor and Capital in Production

Research in Applied Economics
ISSN 1948-5433
2019, Vol. 11, No. 3

$$L_t = \left[\left(\frac{\theta}{\theta-1}\right)\left(\frac{1}{1-\alpha}\right)^{1-\alpha}\left(\frac{1}{\alpha}\right)^{\alpha} W_t^{1-\alpha} r_t^{\alpha}\right]^{-\theta}\left[\frac{W_t\alpha}{r_t(1-\alpha)}\right]^{-\alpha} Z_t^{\theta-1} \left[\tilde{a}_D^{\theta-1} P_t^{\gamma} P_{D,t}^{\theta-\gamma} F_t + \tau_t^{1-\theta}\tilde{a}_{x,t}^{\theta-1} P_t^{*\gamma} P_{X,t}^{\theta-\gamma} F_t^*\right] \quad \text{(B.37)}$$

$$K_t = \left[\left(\frac{\theta}{\theta-1}\right)\left(\frac{1}{1-\alpha}\right)^{1-\alpha}\left(\frac{1}{\alpha}\right)^{\alpha} W_t^{1-\alpha} r_t^{\alpha}\right]^{-\theta}\left[\frac{W_t\alpha}{r_t(1-\alpha)}\right]^{1-\alpha} Z_t^{\theta-1} \left[\tilde{a}_D^{\theta-1} P_t^{\gamma} P_{D,t}^{\theta-\gamma} F_t + \tau_t^{1-\theta}\tilde{a}_{x,t}^{\theta-1} P_t^{*\gamma} P_{X,t}^{\theta-\gamma} F_t^*\right] \quad \text{(B.38)}$$

$$L_t^* = \left[\left(\frac{\theta}{\theta-1}\right)\left(\frac{1}{1-\alpha}\right)^{1-\alpha}\left(\frac{1}{\alpha}\right)^{\alpha} r_t^{*\alpha}\right]^{-\theta}\left[\frac{\alpha}{r_t^*(1-\alpha)}\right]^{-\alpha} Z_t^{*\theta-1} \left[\tilde{a}_D^{\theta-1} P_t^{*\gamma} P_{D,t}^{*\theta-\gamma} F_t^* + \tau_t^{*1-\theta}\tilde{a}_{x,t}^{*\theta-1} P_t^{\gamma} P_{X,t}^{*\theta-\gamma} F_t\right] \quad \text{(B.39)}$$

$$K_t^* = \left[\left(\frac{\theta}{\theta-1}\right)\left(\frac{1}{1-\alpha}\right)^{1-\alpha}\left(\frac{1}{\alpha}\right)^{\alpha} r_t^{*\alpha}\right]^{-\theta}\left[\frac{\alpha}{r_t^*(1-\alpha)}\right]^{1-\alpha} Z_t^{*\theta-1} \left[\tilde{a}_D^{\theta-1} P_t^{*\gamma} P_{D,t}^{*\theta-\gamma} F_t^* + \tau_t^{*1-\theta}\tilde{a}_{x,t}^{*\theta-1} P_t^{\gamma} P_{X,t}^{*\theta-\gamma} F_t\right] \quad \text{(B.40)}$$

Bond Market Equilibrium

$$B_{H,t+1} + B_{H,t+1}^* = 0 \quad \text{(B.41)}$$

$$B_{F,t+1}^* + B_{F,t+1} = 0 \quad \text{(B.42)}$$

Final Goods Resource Constraint

$$F_t = C_t + X_t \quad \text{(B.43)}$$

$$F_t^* = C_t^* + X_t^* \quad \text{(B.44)}$$

Balance of Payments

$$P_t B_{H,t+1} + P_t^* B_{F,t+1} = (1+i_t)P_t B_{H,t} + (1+i_t^*)P_t^* B_{F,t} + \frac{1}{2}(W_t L_t - L_t^*) + \frac{1}{2}(R_t K_t - R_t^* K_t^*) - \frac{1}{2}(P_t C_t - P_t^* C_t^*) - \frac{1}{2}(P_t X_t - P_t^* X_t^*) + \frac{1}{2}\left(N_{D,t}\tilde{d}_t - N_{D,t}^*\tilde{d}_t^*\right) - \frac{1}{2}\left(N_{E,t}\tilde{v}_t - N_{E,t}^*\tilde{v}_t^*\right) \quad \text{(B.45)}$$

Research in Applied Economics
ISSN 1948-5433
2019, Vol. 11, No. 3

Appendix C. Empirical Evidence

I start to plot time series for GDP, consumption, investment and labor for the U.S. over the sample post-Bretton Woods period 1973Q1-2009Q4[1]. The GDP time series display large fluctuations about its trend at shorter frequencies. Consumption, investment and labor time series seem to comove with the GDP series. To make a comparison of the model dynamics Figure 3: Times Series using U.S. data with the cyclical properties of the empirical data, cyclical component of the actual time series are needed to be extracted. As in the analysis of King & Rebelo (1999) and Backus et al. (1992), Hodrick-Prescott (HP) filter[2] by Hodrick and Prescott (1997) with smoothing parameter equal to 1600 to the natural logarithm of each series is applied.

I organize the data into four categories: (1) standard deviations of a variable relative to that of the logarithm of output, (2) correlation between a variable and the logarithm of output as a domestic co-movement, (3) international correlations between home variables and foreign variables, and (4) correlation between relative consumption and a terms of trade as an other correlation. Table 1 provides the U.S. business cycle statistics, 1973Q1-2009Q4 and its correlations with GDP. As is commonly known, investment is about almost 4 times more volatile (3.87) than output and consumption (0.72) and labor (0.58) are less volatile than output. Consumption, investment and labor are pro-cyclical (0.86, 0.89, 0.79, respectively). The terms of trade is defined as the relative price of imports to exports. It is almost 1.5 times more volatile than output and a correlation with output is small (0.06) compare to other variables. Table 2 provides international correlations between the U.S. and the Europe aggregate[3], and the correlation between terms of trade and relative consumption as an other correlation. Cross-country output correlations (0.55) are larger than cross-country consumption correlations (0.42). Standard IRBC models produce higher consumption correlations than output correlations. Investment and labor tend to be positively correlated across countries (0.39 and 0.28, respectively) in the data. The standard models fail to account this feature. Lastly, the terms of trade and the ratio of consumption is negatively linked in the data (-0.35), but standard models have negative correlation or close to zero.

[1] U.S. quarterly data for GDP, consumption, investment is obtained from the International Financial Statistics provided by the International Monetary Fund (http://elibrary-data.imf.org/). The data for labor is obtained from Bureau of Labor Statistics (http://www.bls.gov/) and OECD.StatExtracts (http://stats.oecd.org). More detail about the U.S. time series are found in an appendix.

[2] Time series data consist of a cyclical component (y_t^c) and a trend component (y_t^d). To extract cyclical component, HP filter is used. It is operational by penalizing variations in the second difference of the trend minimizing $\left\{\sum_{t=0}^{T}(y_t - y_t^d)^2 + \lambda \sum_{t=2}^{T-1}((y_{t+1}^d - y_t^d) - (y_t^d - y_{t-1}^d))^2\right\}$.

[3] The quarterly data for the U.S. and Europe are taken from International Financial Statistics. European countries include Austria, Finland, France, Germany, Italy, Sweden, Switzerland and U.K.

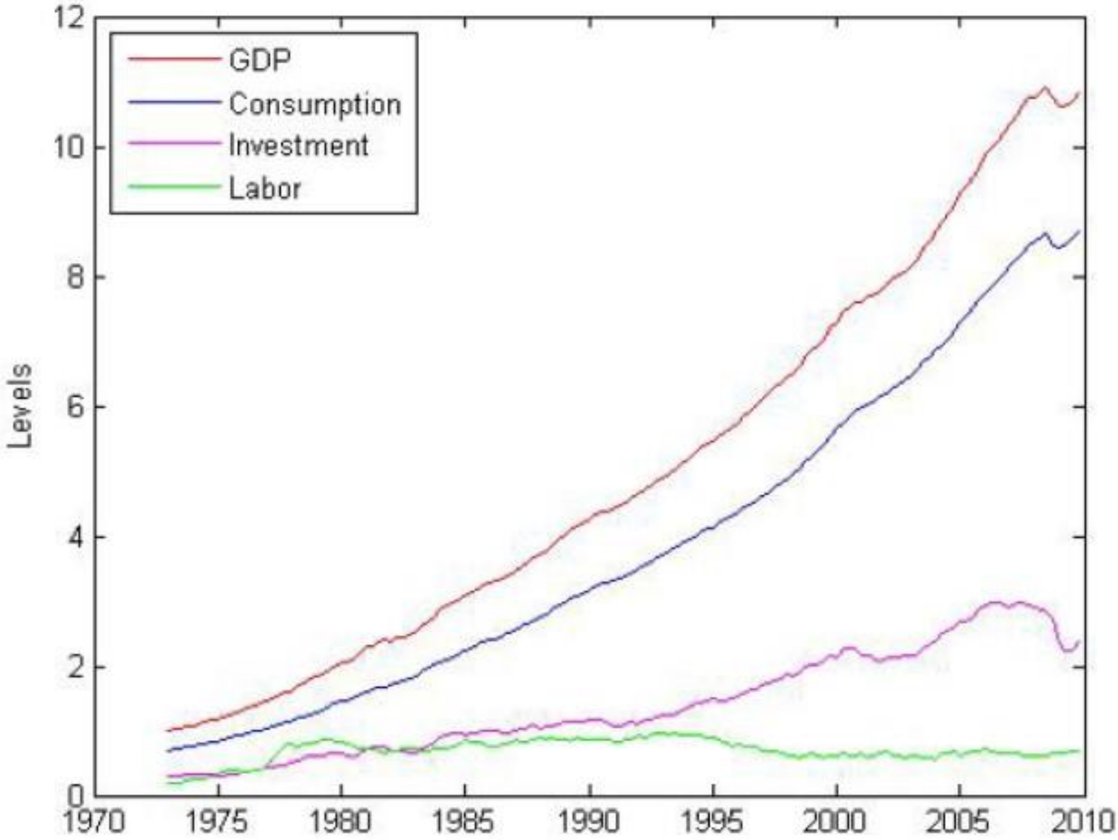

Figure 4. U.S. Data: HP Filtered Trend

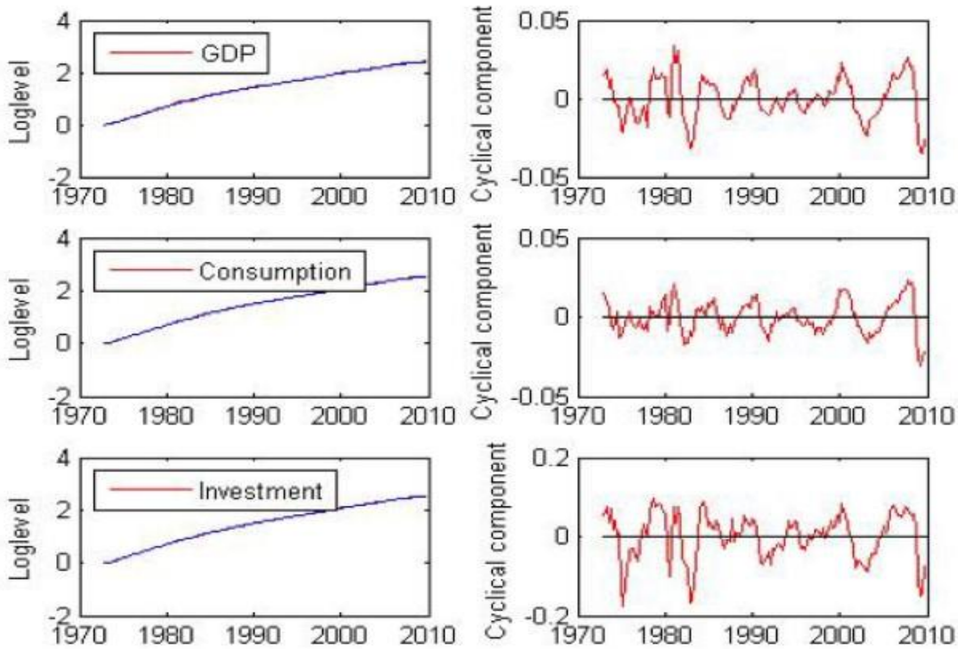

Table 5. U.S. Business Cycle Statistics (1973Q1-2009Q4)

	Volatility	Domestic Comovement
	% S.D. relative to GDP	Correlations with GDP
GDP	1	1
Consumption	0.72	0.86
Investment	3.87	0.89
Employment	0.58	0.79
TOT	1.44	0.06

Table 6. International Correlations and Other Correlation (1973Q1-2008Q3)

GDP, GDP^*	0.55
C, C^*	0.42
X, X^*	0.39
L, L^*	0.28
TOT, Relative Consumption	-0.35 (CDL)