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Analysis of Debt to GDP Ratio with Microeconomic Foundations: Theoretical Basis for MMT Arguments

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Abstract

This paper examines the relationship among budget deficit, inflation rate and debt to GDP ratio from the perspective of Functional Finance Theory and MMT (Modern Monetary Theory). Using an overlapping generations model under monopolistic competition with bequest motive of consumers, mainly we will show the following results.

• Under full employment with constant prices or inflation the debt to GDP ratio does not change from a period to the next period

• The interest rate on government bonds should equal the nominal growth rate to achieve full employment with constant prices or inflation under balanced budget excluding interest payments on government bonds.

• The inflation rate we need to maintain full employment under balanced budget excluding interest payments on government bonds is determined by the interest rate.

Keywords: debt to GDP ratio, overlapping generations model, Functional Finance Theory, MMT

1. Introduction

One of the most commonly used conditions for examining fiscal stability is the Domar condition (Domar(1944), Yoshino, Miyamoto (2020)). The Domar condition compares the interest rate with the economic growth rate under balanced budget (excluding interest payments on the government bonds), and if the former is greater than the latter, public finance will become unstable, and the government debt to GDP ratio will continue to grow. Yoshino, Miyamoto(2020) try to modify the Domar condition by focusing not only on the supply side of government bonds but also on the demand side, while keeping the idea of fiscal instability indicated by the Domar condition. However, our interest is different from that.

Using an overlapping generations model under monopolistic competition according to Otaki(2007, 2009, 2015) with bequest motive of consumers, we will present some results about debt to GDP ratio and the Domar condition from the perspective of Functional Finance Theory (Lerner(1943), Lerner(1944)) and MMT (Modern Money Theory or Modern Monetary Theory, Kelton(2020), Mitchell, Wray, Watts(2019), Wray(2015)(note 1).

In the next section we argue main points of MMT. In Section 3 we present a model. In Section 4 we consider behaviors of consumers and firms. In Section 5 we will show the following main results of this paper.

- (Proposition 1) (1) We need budget deficit (including interest payments on government bonds) to maintain full employment when the economy grows at the positive rate by technological progress under constant prices. (2) If the interest rate on the government bonds is larger than the growth rate, the budget surplus (excluding interest payments) is necessary to maintain full employment under constant prices.

- (Proposition 2) Under full employment with constant prices the debt to GDP ratio does not change from a period to the next period.

- (Proposition 3) The interest rate on government bonds should equal the (real) growth rate to achieve full employment with constant prices under balanced budget excluding interest payments on government bonds.

- (Proposition 4) (1) If the budget deficit is larger than the level which is sufficient to maintain full employment under constant prices, inflation is triggered. (2) If inflation occurs for one period only, then the steady-state value of each variable multiplied by the inflation rate returns the economy to growth under constant prices. (3) If inflation continues, then growth, including price increases, continues under the values of variables that caused inflation multiplied by the inflation rate.

- (Proposition 5) Under full employment with inflation the debt to GDP ratio does not change from a period to the next period.

- (Proposition 6) (1) The interest rate on government bonds should equal the nominal growth rate to achieve full employment with inflation under balanced budget excluding interest payments on government bonds. (2) The inflation rate we need to maintain full employment is determined by the interest rate.

2. A Theoretical Basis to MMT

This paper is one of the attempts to give a theoretical basis to the so-called Functional Finance Theory by Lerner (1943, 1944). It also presents a theoretical and mathematical foundation for MMT. In particular, we provide a rationale for the following claims (Kelton (2020)). We refer to the summary of Kelton's book by Hogan (2021). In fact, Hogan argues that Kelton is wrong, but he summarizes Kelton's argument to the point.

• The treasury creates new money (or governments bonds).

The money supply (or supply of government bonds) equals the savings. Thus, an increase in the money supply equals an increase in the savings. As expressed in the equation (6) in Section 5.2, an increase in the savings equals the budget deficit. The rate of an increase in the savings, which equals the rate of an increase in the money supply, equals the rate of economic growth, and therefore the budget deficit and an increase in the money supply in this case does not cause inflation.

• Inflation is caused by federal government deficit spending, not by Fed policy.

If the actual budget deficit is larger than the budget deficit that is necessary and sufficient to maintain full employment under economic growth, the prices of the goods will rise.

• Federal government spending is not related to taxes or borrowing.

As summarized above, sustained budget deficits are necessary to maintain full employment under economic growth, and these budget deficits make it possible to maintain full employment. It is impossible to maintain full employment in a growing economy with a balanced budget. Therefore, even if the budget deficit to maintain full employment is financed by the national debt, it does not need to be repaid or redeemed, and should not be repaid or redeemed. Future budget surpluses need not and should not make up the deficit for growth.

3. The Model

We consider a two-periods (1: younger or working, and 2: older or retired) overlapping generations (OLG) model under monopolistic competition. Our model is according to Otaki (2007, 2009, 2015), and a generalization of Tanaka (2020) in which perfect competition is assumed. The structure of our model is as follows.

• There is one factor of production, labor, and there is a continuum of perishable goods indexed by $z \in [0,1]$. Good z is monopolistically produced by firm z with increasing or decreasing or constant returns to scale technology. The technology progresses at the rate $\gamma - 1 > 0$.

• During Period 1, each consumer supplies labor, consumes the goods and saves asset and income for his consumption and bequest in Period 2. He is employed or not employed. Savings are made through government bonds that earn interest. More generally, savings can be thought of as being made in terms of interest-bearing government bonds and non-interest-bearing money with endogenously determined interest rate of government bonds, but the results of the

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analysis are similar.

• During Period 2, each consumer consumes the goods using his savings carried over from his Period 1, and he leaves a bequest to the next generation. We assume that the bequest is spent by the next generation consumers in their Period 1, that is, it is spent in Period 2 of the older generation consumers. Therefore, the bequest is not a literal bequest but a gift from the older generation to the younger generation, since it is not used after the death of the previous generation consumers. The bequest is equally distributed to each consumer.

• Each consumer determines his consumptions and bequest in Periods 1 and 2 and the labor supply at the beginning of Period 1 depending on the situation that he is employed or not employed.

We use the following notation.

C_i^e: consumption basket of an employed consumer in Period i, $i = 1,2$.

B^e: Bequest by an employed consumer.

C_i^u: consumption basket of an unemployed consumer in Period i, $i = 1,2$.

B^u: Bequest by an unemployed consumer.

$c_i^e(z)$: consumption of good z of an employed consumer in Period i, $i = 1,2$.

$c_i^u(z)$: consumption of good z of an unemployed consumer in Period i, $i = 1,2$.

P_i: the price of consumption basket in Period i, $i = 1,2$.

$p_i(z)$: the price of good z in Period i, $i = 1,2$.

$\rho = P_2/P_1$: (expected) inflation rate (plus one).

W: nominal wage rate.

Π: profits of firms which are equally distributed to each consumer.

r: interest rate of government bonds.

l: labor supply of an individual.

$\Gamma(l)$: disutility function of labor, which is increasing and convex.

L: employment.

L_f: population of labor or employment in the full employment state.

y: labor productivity, which increases by technological progress. It is increasing or constant or decreasing with respect to "employment $\times$ labor supply" Ll.

$\gamma - 1$: economic growth rate, $\gamma > 1$.

We assume that the population L_f is constant.

We denote a bequest to each consumer by consumers of the previous generation by $\tilde{B}$. We have

$$\tilde{B} = \frac{1}{L_f}\left[L\tilde{B}^e + (L_f - L)\tilde{B}^u\right],\ L_f\tilde{B} = L\tilde{B}^e + (L_f - L)\tilde{B}^u.$$

$\tilde{B}^e$ and $\tilde{B}^u$ are, respectively, bequests by an employed consumer and an unemployed consumer of the previous generation.

4. Behaviors of Agents

4.1 Consumers' Behavior

First we consider utility maximization of consumers. The utility function of employed consumers of one generation over two periods is written as

$$u(C_1^e, C_2^e, \frac{B^e}{P_2}) - \Gamma(l).$$

The utility function of unemployed consumers is

$$u(C_1^u, C_2^u, \frac{B^u}{P_2}).$$

We assume that $u(\cdot,\cdot,\cdot)$ is a homothetic function. The consumption baskets of employed and unemployed consumers in Period i are

$$C_i^e = \left(\int_0^1 c_i^e(z)^{\frac{\sigma-1}{\sigma}} dz\right)^{\frac{\sigma}{\sigma-1}}, i = 1,2,$$

and

$$C_i^u = \left(\int_0^1 c_i^u(z)^{\frac{\sigma-1}{\sigma}} dz\right)^{\frac{\sigma}{\sigma-1}}, i = 1,2.$$

σ is the elasticity of substitution among the goods. $\sigma > 1$. The price of consumption basket in Period i is defined by

$$P_i = \left(\int_0^1 p_i(z)^{1-\sigma} dz\right)^{\frac{1}{1-\sigma}}, i = 1,2.$$

The budget constraint for an employed consumer is

$$\int_0^1 p_1(z)c_1^e(z)dz + \frac{1}{1+r}\int_0^1 p_2(z)c_2^e(z)dz + \frac{1}{1+r}B^e = Wl + \Pi + (1+r)\tilde{B}.$$

The budget constraint for an unemployed consumer is

$$\int_0^1 p_1(z)c_1^u(z)dz + \frac{1}{1+r}\int_0^1 p_2(z)c_2^u(z)dz + \frac{1}{1+r}B^u = \Pi + (1+r)\tilde{B}.$$

Let

$$\alpha=\frac{P_1C_1^e}{P_1C_1^e+\frac{1}{1+r}P_2C_2^e+\frac{1}{1+r}B^e},\beta=\frac{\frac{1}{1+r}P_2C_2^e}{P_1C_1^e+\frac{1}{1+r}P_2C_2^e+\frac{1}{1+r}B^e},\ 1-\alpha-\beta=\frac{\frac{1}{1+r}B^e}{P_1C_1^e+\frac{1}{1+r}P_2C_2^e+B^e}.$$

$0<\alpha<1,\ 0<\beta<1$. Since the utility functions $u(C_1^e,C_2^e,\frac{B^e}{P_2})$ and $u(C_1^u,C_2^u,\frac{B^u}{P_2})$ are homothetic, α and β are determined by the prices, and do not depend on the income of the consumers. Therefore, we have

$$\alpha=\frac{P_1C_1^e}{P_1C_1^e+\frac{1}{1+r}P_2C_2^e+\frac{1}{1+r}B^e}=\frac{P_1C_1^u}{P_1C_1^u+\frac{1}{1+r}P_2C_2^u+\frac{1}{1+r}B^u},$$

$$\beta=\frac{\frac{1}{1+r}P_2C_2^e}{P_1C_1^e+\frac{1}{1+r}P_2C_2^e+\frac{1}{1+r}B^e}=\frac{\frac{1}{1+r}P_2C_2^u}{P_1C_1^u+\frac{1}{1+r}P_2C_2^u+\frac{1}{1+r}B^u}.$$

and

$$1-\alpha-\beta=\frac{B^e}{(1+r)P_1C_1^e+P_2C_2^e+B^e}=\frac{B^u}{(1+r)P_1C_1^u+P_2C_2^u+B^u}.$$

From standard calculations we obtain the following demand functions for consumption baskets and bequests (please see Appendix).

$$C_1^e=\alpha\frac{Wl+\Pi+(1+r)\tilde{B}}{P_1},\ C_2^e=\beta\frac{Wl+\Pi+(1+r)\tilde{B}}{\frac{1}{1+r}P_2},\ \frac{B_2^e}{P_2}=(1-\alpha-\beta)\frac{Wl+\Pi+(1+r)\tilde{B}}{\frac{1}{1+r}P_2},$$

$$C_1^u=\alpha\frac{\Pi+(1+r)\tilde{B}}{P_1},\ C_2^u=\beta\frac{\Pi+(1+r)\tilde{B}}{\frac{1}{1+r}P_2},\ \frac{B^u}{P_2}=(1-\alpha-\beta)\frac{\Pi+(1+r)\tilde{B}}{\frac{1}{1+r}P_2}.$$

Also the following demand functions for good z of employed and unemployed consumers are derived .

$$c_1^e(z)=\left(\frac{p_1(z)}{P_1}\right)^{-\sigma}\frac{\alpha(Wl+\Pi+(1+r)\tilde{B})}{P_1},$$

$$c_2^e(z)=\left(\frac{p_2(z)}{P_2}\right)^{-\sigma}\frac{\beta(Wl+\Pi+(1+r)\tilde{B})}{\frac{1}{1+r}P_2},$$

$$c_1^u(z)=\left(\frac{p_1(z)}{P_1}\right)^{-\sigma}\frac{\alpha(\Pi+(1+r)\tilde{B})}{P_1},$$

and

$$c_2^u(z)=\left(\frac{p_2(z)}{P_2}\right)^{-\sigma}\frac{\beta(\Pi+(1+r)\tilde{B})}{\frac{1}{1+r}P_2}.$$

From these analyses we obtain the indirect utility functions of employed and unemployed consumers as follows:

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$$V^e = u\left(\alpha\frac{Wl+\Pi+(1+r)\tilde{B}}{P_1}, \beta\frac{Wl+\Pi+(1+r)\tilde{B}}{\frac{1}{1+r}P_2}, (1-\alpha-\beta)\frac{Wl+\Pi+(1+r)\tilde{B}}{\frac{1}{1+r}P_2}\right) - \Gamma(l),$$

and

$$V^u = u\left(\alpha\frac{\Pi+(1+r)\tilde{B}}{P_1}, \beta\frac{\Pi+(1+r)\tilde{B}}{\frac{1}{1+r}P_2}, (1-\alpha-\beta)\frac{Wl+\Pi+(1+r)\tilde{B}}{\frac{1}{1+r}P_2}\right).$$

Let

$$\omega = \frac{W}{P_1}.$$

This is the real wage rate. Then, we can write

$$V^e = \varphi\left(\omega l + \frac{\Pi+(1+r)\tilde{B}}{P_1}, \rho, 1+r\right) - \Gamma(l),$$

$$V^u = \varphi\left(\frac{\Pi+(1+r)\tilde{B}}{P_1}, \rho, 1+r\right),$$

Denote

$$I = \omega l + \frac{\Pi+(1+r)\tilde{B}}{P_1}.$$

The condition for maximization of V^e with respect to l given ρ is

$$\frac{\partial\varphi}{\partial I}\omega - \Gamma'(l) = 0, \tag{1}$$

where

$$\frac{\partial\varphi}{\partial I} = \alpha\frac{\partial u}{\partial C_1^e} + \beta\left(\frac{1+r}{\rho}\right)\frac{\partial u}{\partial C_2^e} + (1-\alpha-\beta)\left(\frac{1+r}{\rho}\right)\frac{\partial u}{\partial\left(\frac{B^e}{P_2}\right)}.$$

Given P_1, ρ and r the labor supply is a function of ω. From (1) we get

$$\frac{dl}{d\omega} = \frac{\frac{\partial\varphi}{\partial I} + \frac{\partial^2\varphi}{\partial I^2}\omega l}{\Gamma''(l) - \frac{\partial^2\varphi}{\partial I^2}\omega^2}.$$

If $\frac{dl}{d\omega} > (<) 0$, the labor supply is increasing (decreasing) with respect to the real wage rate ω. But, we assume that the real wage rate does not have a significant effect on the individual labor supply. l may depend on employment L in some way. However, we assume that Ll is increasing with respect to L.

4.2 Firms' Behavior

Let $d_1(z)$ be the total demand for good z by younger generation consumers in Period 1. Then,

$$d_1(z) = \left(\frac{p_1(z)}{P_1}\right)^{-\sigma} \frac{\alpha(WLl+L_f\Pi+L_f(1+r)\tilde{B})}{P_1} = \left(\frac{p_1(z)}{P_1}\right)^{-\sigma} \frac{\alpha(WLl+L_f\Pi+L_f(1+r)\tilde{B})}{P_1}.$$

This is the sum of the demand of employed and unemployed consumers. Similarly, their total demand for good z in Period 2 is written as

$$d_2(z) = \left(\frac{p_2(z)}{P_2}\right)^{-\sigma} \frac{\beta(WLl+L_f\Pi+L_f(1+r)\tilde{B})}{\frac{1}{1+r}P_2}.$$

Let $\overline{d_2(z)}$ be the demand for good z by the older generation. Then,

$$\overline{d_2(z)} = \left(\frac{p_1(z)}{P_1}\right)^{-\sigma} \frac{\beta(\overline{W}\ \overline{L}\overline{l}+L_f\overline{\Pi}+L_f(1+r)\overline{\tilde{B}})}{\frac{1}{1+r}P_1},$$

where $\overline{W}$, $\overline{\Pi}$, $\overline{L}$, $\overline{l}$ and $\overline{\tilde{B}}$ are the nominal wage rate, the profits of firms, the employment, the individual labor supply, the bequest from one more generation before, respectively, during the previous period. In the equilibrium all $p_1(z)$ are equal, and all $p_2(z)$ are equal.

Let

$$M = \beta\left(\overline{W}\ \overline{L}\overline{l} + L_f\overline{\Pi} + L_f(1+r)\overline{\tilde{B}}\right).$$

The total consumption of the older generation consumers is

$$(1+r)M = \beta(1+r)\left(\overline{W}\ \overline{L}\overline{l} + L_f\overline{\Pi} + L_f(1+r)\overline{\tilde{B}}\right).$$

It is the planned consumption that is determined in their Period 1. Their demand for good z is written as $\left(\frac{p_1(z)}{P_1}\right)^{-\sigma}\frac{M}{P_1}$. The total savings (with interest) is

$$(1-\alpha)(1+r)\left(\overline{W}\ \overline{L}\overline{l} + L_f\overline{\Pi} + L_f(1+r)\overline{\tilde{B}}\right).$$

Government expenditure constitutes the national income as well as the consumptions of the younger and older generations. Then, the total demand for good z is written as

$$d(z) = \left(\frac{p_1(z)}{P_1}\right)^{-\sigma}\frac{Y}{P_1}, \tag{2}$$

where Y is the effective demand defined by

$$Y = \alpha(WLl + L_f\Pi + L_f(1+r)\tilde{B}) + G + (1+r)M.$$

G is the government expenditure. The government determines its demand for good z, $g(z)$, to maximize the following index.

$$\left(\int_0^1 g(z)^{\frac{\sigma-1}{\sigma}} dz\right)^{\frac{\sigma}{\sigma-1}},$$

subject to the constraint:

$$\int_0^1 p_1(z)g(z)dz = G.$$

Let L and Ll be employment and the "employment × labor supply" of firm z. The total employment and the total "employment × labor supply" are

$$\int_0^1 Ldz = L,\ \int_0^1 Lldz = Ll.$$

The output of firm z is Lly. By increasing or constant or decreasing returns to scale y is a function of Ll. In the equilibrium $Lly = d(z)$. Then, we have

$$\frac{\partial d(z)}{\partial Ll} = y + Lly'(Ll).$$

In a case of constant returns to scale

$$\frac{\partial d(z)}{\partial Ll} = y.$$

From (2)

$$\frac{\partial p_1(z)}{\partial d(z)} = -\frac{p_1(z)}{\sigma d(z)}.$$

Thus

$$\frac{\partial p_1(z)}{\partial Ll} = -\frac{p_1(z)[y+Lly'(Ll)]}{\sigma d(z)} = -\frac{p_1(z)}{\sigma Ll}\left[1 + \frac{Lly'(Ll)}{y}\right].$$

Define the elasticity of the labor productivity by

$$\zeta = \frac{Lly'(Ll)}{y}.$$

Then,

$$\frac{\partial p_1(z)}{\partial Ll} = -\frac{p_1(z)(1+\zeta)}{\sigma Ll}$$

We assume that ζ is constant and $1 + \zeta > 0$. For increasing (or decreasing) returns to scale technology $\zeta > 0$ (or $(\zeta < 0)$).

The profit of firm z is

$$\pi(z) = p_1(z)Lly - LlW.$$

The condition for profit maximization is

$$\frac{\partial\pi(z)}{\partial Ll} = \left[p_1(z)y - Lly\frac{p_1(z)}{\sigma Ll}\right](1+\zeta) - W = \left[p_1(z)y - \frac{p_1(z)y}{\sigma}\right](1+\zeta) - W = 0.$$

Therefore, we obtain

$$p_1(z) = \frac{1}{\left(1-\frac{1}{\sigma}\right)(1+\zeta)y}W.$$

Let $\mu = 1/\sigma$. Then,

$$p_1(z) = \frac{1}{(1-\mu)(1+\zeta)y}W.$$

This means that the real wage rate is

$$\omega = (1-\mu)(1+\zeta)y.$$

Since all firms are symmetric,

$$P_1 = p_1(z) = \frac{1}{(1-\mu)(1+\zeta)y}W.$$

5. Budget Deficit and Debt to GDP Ratio

5.1 Market Equilibrium

The (nominal) aggregate supply of the goods is equal to

$$WL + L_f\Pi = P_1Lly.$$

The (nominal) aggregate demand is

$$\alpha(WL + L_f\Pi + L_f(1+r)\tilde{B}) + G + (1+r) = \alpha P_1Lly + \alpha L_f(1+r)\tilde{B} + G + (1+r)M.$$

Since they are equal,

$$P_1Lly = \alpha P_1Lly + \alpha L_f(1+r)\tilde{B} + G + (1+r)M. \tag{3}$$

In real terms

$$Lly = \frac{G+(1+r)M+\alpha L_f(1+r)\tilde{B}}{(1-\alpha)P_1}.$$

The equilibrium value of Ll cannot be larger than $L_f l(L_f)$. $l(L_f)$ is the labor supply when full employment is achieved. However, Ll may be strictly smaller than $L_f l(L_f)$. Then, we have $L < L_f$ and involuntary unemployment exists. If the government collects a tax T from the younger generation consumers, (3) is rewritten as

$$P_1 Lly = \alpha(P_1 Lly + L_f(1+r)\tilde{B} - T) + G + M.$$

5.2 Budget Deficit to Maintain Full Employment

Suppose that up to Period t full employment has been achieved under constant prices. Then, the following equation holds.

$$P_1^t L_f l(L_f) y = \alpha(P_1^t L_f l(L_f) y + L_f(1+r)\tilde{B}^{t-1} - T^t) + G^t + (1+r)M^{t-1}. \quad (4)$$

Superscript t represents the values in Period t. M^{t-1} is the savings, and $\tilde{B}^{t-1}$ is the bequest of the previous generation consumers. The savings of the younger generation consumers is

$$M^t + L_f\tilde{B}^t = (1-\alpha)\left(P_1^t L_f l\left(L_f\right) y + L_f(1+r)\tilde{B}^{t-1} - T^t\right) \quad (5)$$

$$= G^t + L_f(1+r)\tilde{B}^{t-1} - T^t + (1+r)M^{t-1}.$$

Their consumptions in their Period 2 and the bequests are, respectively,

$$\beta(1+r)(P_1^t L_f l(L_f) y + L_f(1+r)\tilde{B}^{t-1} - T^t)$$
$$= \frac{\beta}{1-\alpha}(1+r)[G^t + L_f(1+r)\tilde{B}^{t-1} - T^t + (1+r)M^{t-1}],$$

and

$$(1-\alpha-\beta)(1+r)(P_1^t L_f l(L_f) y + L_f(1+r)\tilde{B}^{t-1} - T^t)$$
$$= \frac{1-\alpha-\beta}{1-\alpha}(1+r)[G^t + L_f(1+r)\tilde{B}^{t-1} - T^t + (1+r)M^{t-1}].$$

In order to maintain full employment under growth by technological progress (5) must be equal to $\gamma(M^{t-1} + L_f\tilde{B}^{t-1})$. Therefore, we obtain

$$G^t - T^t = (\gamma - 1 - r)(M^t + L_f\tilde{B}^{t-1}),$$

or

$$G^t - T^t + r(M^{t-1} + L_f\tilde{B}^{t-1}) = (\gamma - 1)(M^{t-1} + L_f\tilde{B}^{t-1}). \quad (6)$$

The left hand-side is the budget deficit including interest payments on government bonds. Since $M^{t-1} + L_f\tilde{B}^{t-1}$ is positive, $G^t - T^t + r(M^{t-1} + L_f\tilde{B}^{t-1}) > 0$ when $\gamma > 1$. In Period $t +$

1 $M^t = \gamma M^{t-1}$, $\tilde{B}^t = \gamma \tilde{B}^{t-1}$, and we can assume $G^{t+1} = \gamma G^t$ and $T^{t+1} = \gamma T^t$. Thus, with $P_1^{t+1} = P_1^t$ we obtain

$$P_1^t L_f l(L_f)\gamma y = \alpha(P_1^t L_f l(L_f)\gamma y - \gamma T^t + \gamma L_f(1+r)\tilde{B}^{t-1}) + \gamma G^t + \gamma(1+r)M^{t-1}.$$

This is equivalent to (4), and full employment is maintained by $G^{t+1} = \gamma G^t$ and $T^{t+1} = \gamma T^t$.

Budget deficit is necessary under growth because of deficiency of the savings of the older generation. (6) means that an increase in the savings from Period t to the next period equals the budget deficit including interest payments on government bonds. (6) does not holds when the left-hand side is zero or negative, that is, under budget surplus or balanced budget including interest payments full employment is not realized in a growing economy with constant prices. However, if $r > \gamma - 1$, that is the interest rate on government bonds is larger than the growth rate, we need budget surplus $(G^t - T^t < 0)$ excluding interest payments. Summarizing the results.

Proposition 1 (1) We need budget deficit (including interest payments on government bonds) to maintain full employment when the economy grows at the positive rate by technological progress under constant prices. (2) If the interest rate on the government bonds is larger than the growth rate, the budget surplus excluding interest payments is necessary to maintain full employment under constant prices.

5.3 Debt to GDP Ratio under Full Employment with Constant Prices

The debt to GDP ratio in Period t is $\frac{M^t + L_f\tilde{B}^t}{P_1 L l y}$. In which the debt at the end of Period t is

$$M^t + L_f\tilde{B}^t.$$

From

$$M^t + L_f\tilde{B}^t = \gamma(M^{t-1} + L_f\tilde{B}^{t-1}),$$

we have

$$\frac{M^t + L_f\tilde{B}^t}{P_1 L l y} = \frac{M^{t-1} + L_f\tilde{B}^{t-1}}{\frac{P_1 L l y}{\gamma}}.$$

$\frac{P_1 L l y}{\gamma}$ is GDP in Period $t-1$. Therefore, the debt to GDP ratio does not change if full employment is achieved under constant prices.

Proposition 2 Under full employment with constant prices the debt to GDP ratio does not change from a period to the next period.

Domar condition

The so-called Domar condition (Domar(1944)) says that if the interest rate on the government bonds is larger than the growth rate under balanced budget excluding interest payments (so-called primary balance), the debt to GDP ratio diverges to infinity. From (6), if $G^t = T^t$, we have

$$r = \gamma - 1. \tag{7}$$

This means that the interest rate should be equal the growth rate under full employment with constant prices. If the interest rate is larger than the (real) growth rate, that is $r > \gamma - 1$, the prices cannot be constant. Please see Section 5.6. On the other hand, if the interest rate is smaller than the (real) growth rate, that is, $r < \gamma - 1$, aggregate demand is insufficient, and recession will occur. Summarizing the results,

Proposition 3 The interest rate on government bonds should equal the (real) growth rate to achieve full employment with constant prices under balanced budget excluding interest payments on government bonds.

5.5 Excessive Budget Deficit and Inflation

Suppose that up to Period $t-1$ full employment has been achieved under constant prices, but the government expenditure and/or the tax in Period t may be different from their steady state values. The steady state is a state such that full employment is continuously maintained under constant prices. Denote them by $\hat{G}^t$ and $\hat{T}^t$. We denote also the actual price by $\hat{P}_1^t$. Then, the following equation holds.

$$\hat{P}_1^t L_f l(L_f) y = \alpha(\hat{P}_1^t L_f l(L_f) y + L_f(1+r)\tilde{B}^{t-1} - \hat{T}^t) + \hat{G}^t + (1+r)M^{t-1}. \tag{8}$$

The savings of the younger generation consumers is

$$M^t + L_f \tilde{B}^t = (1-\alpha)(\hat{P}_1^t L_f l(L_f) y + L_f(1+r)\tilde{B}^{t-1} - \hat{T}^t) \tag{9}$$

$$= \hat{G}^t + L_f(1+r)\tilde{B}^{t-1} - \hat{T}^t + (1+r)M^{t-1}.$$

Their consumptions in their Period 2 and the bequests are, respectively,

$$\beta(1+r)(\hat{P}_1^t L_f l(L_f) y + L_f(1+r)\tilde{B}^{t-1} - \hat{T}^t)$$
$$= \frac{\beta}{1-\alpha}(1+r)(\hat{G}^t + L_f(1+r)\tilde{B}^{t-1} - \hat{T}^t + (1+r)M^{t-1}),$$

and

$$(1-\alpha-\beta)(1+r)(\hat{P}_1^t L_f l(L_f) y + L_f \tilde{B}^{t-1} - \hat{T}^t)$$
$$= \frac{1-\alpha-\beta}{1-\alpha}(1+r)(\hat{G}^t + L_f(1+r)\tilde{B}^{t-1} - \hat{T}^t + (1+r)M^{t-1}).$$

Let

$$\rho = \frac{\hat{P}_1^t}{P_1^t} > 1.$$

If in Period $t+1$ full employment is maintained with $P_1^{t+1} = \hat{P}_1^t > P_1^t$, (9) must be equal to $\gamma\rho(M^{t-1} + L_f\tilde{B}^{t-1})$. Thus,

$$\hat{G}^t - \hat{T}^t = (\gamma\rho - 1 - r)(M^{t-1} + L_f\tilde{B}^{t-1}),$$

or

$$\hat{G}^t - \hat{T}^t + r(M^{t-1} + L_f\tilde{B}^{t-1}) = (\gamma\rho - 1)(M^{t-1} + L_f\tilde{B}^{t-1}). \tag{10}$$

Then, from (6)

$$\hat{G}^t - \hat{T}^t + r(M^{t-1} + L_f\tilde{B}^{t-1}) = (\gamma\rho - 1)(M^{t-1} + L_f\tilde{B}^{t-1}) > (\gamma - 1)(M^{t-1} + L_f\tilde{B}^{t-1})$$

$$= G^t - T^t + r(M^{t-1} + L_f\tilde{B}^{t-1}).$$

This means

$$(\zeta - 1)\rho(M^{t-1} + L_f\tilde{B}^{t-1}) = \hat{G}^t - \hat{T}^t - (G^t - T^t),$$

or

$$\rho - 1 = \frac{\hat{G}^t - \hat{T}^t - (G^t - T^t)}{\gamma(M^{t-1} + L_f\tilde{B}^{t-1})} > 0.$$

Therefore, excessive budget deficit, $\hat{G}^t - \hat{T}^t - (G^t - T^t)$, causes inflation at the rate of $\frac{\hat{P}_1^t}{P_1^t}$. In Period $t+1$, $M^{t+1} = \gamma\rho M^t$, $\tilde{B}^{t+1} = \gamma\rho\tilde{B}^t$, and we can assume $G^{t+1} = \gamma\rho G^t$ and $T^{t+1} = \gamma\rho T^t$. Thus, with $P_1^{t+1} = \hat{P}_1^t$ we obtain

$$\hat{P}_1^t L_f l(L_f)\gamma y = \alpha(\hat{P}_1^t L_f l(L_f)\gamma y + \gamma\rho L_f(1+r)\tilde{B}^{t-1} - \gamma\rho T^t) + \gamma\rho G^t + \gamma\rho(1+r)M^{t-1}.$$

Since $\hat{P}_1^t = \rho P_1^t$, this is equivalent to (4), and full employment is maintained by $G^{t+1} = \gamma\rho G^t$ and $T^{t+1} = \gamma\rho T^t$. In this case, inflation occurs for one period only, and the steady-state value of each variable multiplied by the inflation rate returns the economy to growth under constant prices.

On the other hand, suppose that in Period $t+1$ $P_1^{t+1} = \rho\hat{P}_1^t$, that is, inflation continues. Then, the following equation holds.

$$\hat{P}_1^t L_f l(L_f)\gamma\rho y = \alpha(\hat{P}_1^t L_f l(L_f)\gamma\rho y + \gamma\rho L_f(1+r)\tilde{B}^{t-1} - \gamma\rho\hat{T}^t) + \gamma\rho\hat{G}^t + \gamma\rho(1+r)M^{t-1}.$$

This is equivalent to (8), and full employment is maintained . In this case, inflation continues and growth, including price increases, continues under the value of the variables that caused inflation multiplied by the inflation rate.

Summarizing the results.

Proposition 4 (1) If the budget deficit is larger than the level which is sufficient to maintain full employment under constant prices, inflation is triggered. (2) If inflation occurs for one period only, then the steady-state value of each variable multiplied by the inflation rate returns the economy to growth under constant prices. (3) If inflation continues, then growth, including price increases, continues under the values of variables that caused inflation multiplied by the inflation rate.

5.6 Debt to GDP Ratio under Full Employment when Inflation Continues, and the Domar Condition

The debt to GDP ratio in Period t is $\frac{M^{t-1}+L_f\tilde{B}^{t-1}}{P_1Lly}$. From $M^t+L_f\tilde{B}^t=\gamma\rho(M^{t-1}+L_f\tilde{B}^{t-1})$, we have

$$\frac{M^t+L_f\tilde{B}^t}{P_1Lly}=\frac{M^{t-1}+L_f\tilde{B}^{t-1}}{\frac{P_1Lly}{\gamma\rho}}.$$

Therefore, the debt to GDP ratio does not change if full employment is maintained under inflation. This relation holds in both the one period inflation case and the continued inflation case.

Proposition 5 Under full employment with inflation the debt to GDP ratio does not change from a period to the next period.

Domar condition

From (10), if $\hat{G}^t=\hat{T}^t$, we have

$$r=\gamma\zeta-1. \tag{11}$$

$\gamma\zeta-1$ is the nominal growth rate. This equation means that the interest rate should equal the nominal growth rate under full employment with inflation. In (11) the values of r and γ are given. Thus, this determines the value of ζ, that is, the interest rate with the real growth rate determines the value of inflation rate which we need to maintain full employment. From (11) we have

$$\zeta=\frac{1+r}{\gamma}.$$

Therefore, we need inflation to maintain full employment if the interest rate is larger than the real growth rate $\gamma-1$. The value of the inflation rate is determined by the interest rate given the real growth rate. Summarizing the results,

Proposition 6 (1) The interest rate on government bonds should equal the nominal growth rate to achieve full employment with inflation under balanced budget excluding interest payments on government bonds. (2) The inflation rate we need to maintain full employment under balanced budget excluding interest payments on government bonds is determined by the interest rate.

6. Concluding Remark

Using an overlapping generations model under monopolistic competition with bequest motive of consumers we have examined the relationship among budget deficit, inflation rate and debt to GDP ratio from the perspective of Functional Finance Theory and MMT. The main results are as follows.

• Under full employment with constant prices or inflation the debt to GDP ratio does not change from a period to the next period.

• The interest rate on government bonds should equal the nominal growth rate to achieve full employment with inflation or constant prices under balanced budget excluding interest payments on government bonds.

• The inflation rate we need to maintain full employment under balanced budget excluding interest payments on government bonds is determined by the interest rate.

In this paper, the real economic growth rate was assumed to be exogenous, but in the future, we would like to endogenize the growth rate and deal with cases in which fiscal policy affects the growth rate.

Appendix

The consumption baskets and their prices are

$$C_1^e=\left(\int_0^1 c_1^e(z)^{\frac{\sigma-1}{\sigma}}dz\right)^{\frac{\sigma}{\sigma-1}},\ C_2^e=\left(\int_0^1 c_2^e(z)^{\frac{\sigma-1}{\sigma}}dz\right)^{\frac{\sigma}{\sigma-1}},$$

$$C_1^u=\left(\int_0^1 c_1^u(z)^{\frac{\sigma-1}{\sigma}}dz\right)^{\frac{\sigma}{\sigma-1}},\ C_2^u=\left(\int_0^1 c_2^u(z)^{\frac{\sigma-1}{\sigma}}dz\right)^{\frac{\sigma}{\sigma-1}},$$

$$P_1=\left(\int_0^1 p_1(z)^{1-\sigma}dz\right)^{\frac{1}{1-\sigma}},$$

and

$$P_2=\left(\int_0^1 p_2(z)^{1-\sigma}dz\right)^{\frac{1}{1-\sigma}}.$$

The budget constraint for an employed consumer is

$$\int_0^1 p_1(z)c_1^e(z)dz + \frac{1}{1+r}\int_0^1 p_2(z)c_2^e(z)dz + \frac{1}{1+r}B^e = Wl + \Pi + (1+r)\tilde{B}.$$

The budget constraint for an unemployed consumer is

$$\int_0^1 p_1(z)c_1^u(z)dz + \frac{1}{1+r}\int_0^1 p_2(z)c_2^u(z)dz + \frac{1}{1+r}B^u = \Pi + (1+r)\tilde{B}.$$

The Lagrange functions for them are

$$\mathcal{L}^e = u\left(C_1^e, C_2^e, \frac{B^e}{P_2}\right) - \lambda\left(\int_0^q p_1(z)c_1^e(z)dz + \frac{1}{1+r}\int_0^q p_2(z)c_2^e(z)dz + \frac{1}{1+r}B^e - Wl - \Pi - (1+r)\tilde{B}\right),$$

and

$$\mathcal{L}^u = u\left(C_1^u, C_2^u, \frac{B^u}{P_2}\right) - \lambda\left(\int_0^q p_1(z)c_1^u(z)dz + \frac{1}{1+r}\int_0^q p_2(z)c_2^u(z)dz + \frac{1}{1+r}B^u - \Pi - (1+r)\tilde{B}\right).$$

We consider utility maximization for an employed consumer. The first order conditions about consumptions are

$$\frac{\partial u(C_1^e, C_2^e, \frac{B^e}{P_2})}{\partial C_1^e}\left(\int_0^1 c_1^e(z)^{\frac{\sigma-1}{\sigma}}dz\right)^{\frac{1}{\sigma-1}} c_1^e(z)^{-\frac{1}{\sigma}} - \lambda p_1(z) = 0, \quad \text{(A.1)}$$

and

$$\frac{\partial u(C_1^e, C_2^e, \frac{B^e}{P_2})}{\partial C_2^e}\left(\int_0^1 c_2^e(z)^{\frac{\sigma-1}{\sigma}}dz\right)^{\frac{1}{\sigma-1}} c_2^e(z)^{-\frac{1}{\sigma}} - \lambda \frac{1}{1+r}p_2(z) = 0. \quad \text{(A.2)}$$

From them we obtain

$$\frac{\partial u(C_1^e, C_2^e, \frac{B^e}{P_2})}{\partial C_1^e}\left(\int_0^1 c_1^e(z)^{\frac{\sigma-1}{\sigma}}dz\right)^{\frac{1}{\sigma-1}} \int_0^1 c_1^e(z)^{\frac{\sigma-1}{\sigma}}dz - \lambda\int_0^1 p_1(z)c_1^e(z)dz = 0,$$

and

$$\frac{\partial u(C_1^e, C_2^e, \frac{B^e}{P_2})}{\partial C_2^e}\left(\int_0^1 c_2^e(z)^{\frac{\sigma-1}{\sigma}}dz\right)^{\frac{1}{\sigma-1}} \int_0^1 c_2^e(z)^{\frac{\sigma-1}{\sigma}}dz - \lambda\frac{1}{1+r}\int_0^1 p_2(z)c_2^e(z)dz = 0.$$

Then, we get

$$\frac{\partial u(C_1^e,C_2^e,\frac{B^e}{P_2})}{\partial C_1^e}C_1^e-\lambda\int_0^1 p_1(z)c_1^e(z)dz=0, \tag{A.3}$$

and

$$\frac{\partial u(C_1^e,C_2^e,\frac{B^e}{P_2})}{\partial C_2^e}C_2^e-\lambda\frac{1}{1+r}\int_0^1 p_2(z)c_2^e(z)dz=0. \tag{A.4}$$

On the other hand, from (A.1) and (A.2) we have

$$\left(\frac{\partial u\left(C_1^e,C_2^e,\frac{B^e}{P_2}\right)}{\partial C_1^e}\right)^{1-\sigma}\left(\int_0^1 c_1^e(z)^{\frac{\sigma-1}{\sigma}}dz\right)^{-1}\int_0^1 c_1^e(z)^{\frac{\sigma-1}{\sigma}}dz-\lambda^{1-\sigma}\int_0^1 p_1(z)^{1-\sigma}dz=0,$$

and

$$\left(\frac{\partial u(C_1^e,C_2^e,\frac{B^e}{P_2})}{\partial C_2^e}\right)^{1-\sigma}\left(\int_0^1 c_2^e(z)^{\frac{\sigma-1}{\sigma}}dz\right)^{-1}\int_0^1 c_2^e(z)^{\frac{\sigma-1}{\sigma}}dz-\lambda^{1-\sigma}\left(\frac{1}{1+r}\right)^{1-\sigma}\int_0^1 p_2(z)^{1-\sigma}dz=0.$$

Thus, we obtain

$$\frac{\partial u(C_1^e,C_2^e,\frac{B^e}{P_2})}{\partial C_1^e}-\lambda\left(\int_0^1 p_1(z)^{1-\sigma}dz\right)^{\frac{1}{1-\sigma}}=0,$$

and

$$\frac{\partial u(C_1^e,C_2^e,\frac{B^e}{P_2})}{\partial C_2^e}-\lambda\frac{1}{1+r}\left(\int_0^1 p_2(z)^{1-\sigma}dz\right)^{\frac{1}{1-\sigma}}=0.$$

They are written as

$$\frac{\partial u(C_1^e,C_2^e,\frac{B^e}{P_2})}{\partial C_1^e}=\lambda P_1,\quad \frac{\partial u(C_1^e,C_2^e,\frac{B^e}{P_2})}{\partial C_2^e}=\lambda\frac{1}{1+r}P_2. \tag{A.5}$$

Further, from (A.3) and (A.4) we get

$$P_1C_1^e=\int_0^1 p_1(z)c_1^e(z)dz,\quad P_2C_2^e=\int_0^1 p_2(z)c_2^e(z)dz.$$

By the budget constraint

$$P_1C_1^e+\frac{1}{1+r}P_2C_2^e+B^e=Wl+\Pi+(1+r)\tilde{B}. \tag{A.6}$$

The equations in (A.5) are the conditions for maximization of $u\left(C_1^e,C_2^e,\frac{B^e}{P_2}\right)$ about

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C_1^e and C_2^e subject to (A.6). The condition for maximization of $u\left(C_1^e, C_2^e, \frac{B^e}{P_2}\right)$ about the bequest is

$$\frac{1}{P_2}\frac{\partial u(C_1^e, C_2^e, \frac{B^e}{P_2})}{\partial B^e} = \lambda\frac{1}{1+r}.$$

This means

$$\frac{\partial u(C_1^e, C_2^e, \frac{B^e}{P_2})}{\partial B^e} = \lambda\frac{1}{1+r}P_2.$$

We can show similar results for an unemployed consumer. Since $u\left(C_1^e, C_2^e, \frac{B^e}{P_2}\right)$ and $u(C_1^u, C_2^u, \frac{B^u}{P_2})$ are homothetic,

$$\alpha = \frac{P_1C_1^e}{P_1C_1^e + \frac{1}{1+r}P_2C_2^e + \frac{1}{1+r}B^e},\ \beta = \frac{P_2C_2^e}{P_1C_1^e + \frac{1}{1+r}P_2C_2^e + \frac{1}{1+r}B^e}$$

are determined by the prices, and do not depend on the income of consumers. Therefore,

$$\alpha = \frac{P_1C_1^e}{P_1C_1^e + \frac{1}{1+r}P_2C_2^e + \frac{1}{1+r}B^e} = \frac{P_1C_1^u}{(1+r)P_1C_1^u + \frac{1}{1+r}P_2C_2^u + \frac{1}{1+r}B^u},$$

$$\beta = \frac{\frac{1}{1+r}P_2C_2^e}{P_1C_1^e + \frac{1}{1+r}P_2C_2^e + \frac{1}{1+r}B^e} = \frac{\frac{1}{1+r}P_2C_2^u}{P_1C_1^u + \frac{1}{1+r}P_2C_2^u + \frac{1}{1+r}B^u}$$

hold. Also we have

$$1-\alpha-\beta = \frac{\frac{1}{1+r}B^e}{P_1C_1^e + \frac{1}{1+r}P_2C_2^e + \frac{1}{1+r}B^e} = \frac{\frac{1}{1+r}B^u}{P_1C_1^u + \frac{1}{1+r}P_2C_2^u y + \frac{1}{1+r}B^u}$$

From these analyses we obtain the demand functions for consumption baskets as follows.

$$C_1^e = \frac{\alpha(Wl+\Pi+(1+r)\tilde{B})}{P_1},\ C_2^e = \frac{\beta(Wl+\Pi+(1+r)\tilde{B})}{\frac{1}{1+r}P_2} \tag{A.7}$$

and

$$C_1^u = \frac{\alpha(\Pi+(1+r)\tilde{B})}{P_1},\ C_2^u = \frac{\beta(\Pi+(1+r)\tilde{B})}{\frac{1}{1+r}P_2} \tag{A.8}$$

Also we have

$$\frac{B_2^e}{P_2} = (1-\alpha-\beta)\frac{Wl+\Pi+(1+r)\tilde{B}}{\frac{1}{1+r}P_2},\ \frac{B_2^u}{P_2} = (1-\alpha-\beta)\frac{\Pi+(1+r)\tilde{B}}{\frac{1}{1+r}P_2},$$

or

$$B_2^e = (1-\alpha-\beta)(Wl+\Pi+(1+r)\tilde{B}),\ B_2^u = (1-\alpha-\beta)(\Pi+(1+r)\tilde{B}).$$

From (A.1), (A.2), (A.5)

$$P_1\left(\int_0^1 c_1^e(z)^{\frac{\sigma-1}{\sigma}}dz\right)^{\frac{1}{\sigma-1}} c_1^e(z)^{-\frac{1}{\sigma}} - p_1(z) = 0,$$

$$P_2\left(\int_0^1 c_2^e(z)^{\frac{\sigma-1}{\sigma}}dz\right)^{\frac{1}{\sigma-1}} c_2^e(z)^{-\frac{1}{\sigma}} - \frac{1}{1+r}p_2(z) = 0.$$

This means

$$P_1^{-\sigma}\left(\int_0^1 c_1^e(z)^{\frac{\sigma-1}{\sigma}}dz\right)^{-\frac{\sigma}{\sigma-1}} c_1^e(z) - p_1(z)^{-\sigma} = 0,$$

$$P_2^{-\sigma}\left(\int_0^1 c_2^e(z)^{\frac{\sigma-1}{\sigma}}dz\right)^{-\frac{\sigma}{\sigma-1}} c_2^e(z) - \frac{1}{1+r}p_2(z)^{-\sigma} = 0.$$

Thus, we have

$$P_1^{-\sigma}\frac{1}{C_1^e}c_1^e(z) - p_1(z)^{-\sigma} = 0,$$

and

$$P_2^{-\sigma}\frac{1}{C_2^e}c_2^e(z) - \frac{1}{1+r}p_2(z)^{-\sigma} = 0.$$

From them and (A.7) we get the demand function for good z as follows.

$$c_1^e(z) = \left(\frac{p_1(z)}{P_1}\right)^{-\sigma}\frac{\alpha(Wl+\Pi+(1+r)\tilde{B})}{P_1},$$

$$c_2^e(z) = \left(\frac{p_2(z)}{P_2}\right)^{-\sigma}\frac{\beta(Wl+\Pi+(1+r)\tilde{B})}{\frac{1}{1+r}P_2}.$$

Similarly, for an unemployed consumer by (A.8)

$$c_1^u(z) = \left(\frac{p_1(z)}{P_1}\right)^{-\sigma}\frac{\alpha(\Pi+(1+r)\tilde{B})}{P_1},$$

and

$$c_2^u(z) = \left(\frac{p_2(z)}{P_2}\right)^{-\sigma}\frac{\beta(\Pi+(1+r)\tilde{B})}{\frac{1}{1+r}P_2}$$

are obtained. In the equilibrium all $p_1(z)$ are equal, and all $p_2(z)$ are equal.

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Note

Note 1. Japanese references of MMT are Mochizuki(2020), Morinaga(2020), Nakano(2020), Park(2020), Shimakura(2019).

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