Cost-Volume-Profit Analysis for a Multi-Product Company: Micro Approach

Dr. Seung Hwan Kim
Associate Professor, Indiana University of Pennsylvania
United States
E-mail: seung.kim@iup.edu

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Abstract
Cost-volume-profit (CVP) analysis is one of the most common-and-important chapters in an introductory managerial accounting course. While a CVP analysis for a single-product company is relatively easier to be illustrated, the CVP analysis for a multi-product company necessarily takes extra steps to illustrate. For the case of a multi-product company having a sales mix ratio among their products, this study developed a micro approach to the handling of decimals, if appearing, when the company finds their break-even point and target profit point. This study exemplifies how the developed approach gets to closer answers to a break-even point and a target profit point than an existing approach.

Keywords: Cost-volume-profit analysis, CVP analysis, multiple products, sales mix, break-even point, and target profit.
Introduction

In a CVP analysis of a company that sells single or multiple products, a break-even point and a target profit point are found for the single product, or for the multiple products given the sales mix ratio among the products. The focus of the study is on a company that sells multiple products, and the micro focus of the study is on how to handle decimals if they appear when finding a break-even point and a target profit point for the company.

Next presented is the direct comparison between the current approach and suggested approach of the study to finding a break-even point and a target profit point for a multi-product company

Comparison between Current Approach and Suggested Approach

In order to illustrate the current approach and, also, the suggested approach of the study to decimal rounding, a three-product company is created with the following selling prices and other related cost data.

Table 1. A Three-Product Company

<table>
<thead>
<tr>
<th>Product</th>
<th>Product A</th>
<th>Product B</th>
<th>Product C</th>
</tr>
</thead>
<tbody>
<tr>
<td>Unit Selling Price</td>
<td>$30</td>
<td>$42</td>
<td>$62</td>
</tr>
<tr>
<td>Unit Variable Costs</td>
<td>$10</td>
<td>$17</td>
<td>$25</td>
</tr>
<tr>
<td>Unit Contribution Margin</td>
<td>$20</td>
<td>$25</td>
<td>$37</td>
</tr>
<tr>
<td>Sales Mix Ratio</td>
<td>4</td>
<td>3</td>
<td>1</td>
</tr>
</tbody>
</table>

Fixed Costs for the Period = $1,656
Target Profit for the Period = $7,440

Break-Even Point

Current Approach

Under the current approach, the break-even point of the company is calculated as follows:

Step 1: \((20\times4) + (25\times3) + (37\times1) = 80 + 75 + 37 = 192\)

Step 2: \(192 ÷ (4+3+1) = 192 ÷ 8 = 24\)

$24 being the average contribution margin by the sales mix

Step 3: \(X\) being the total number of units to be sold to break-even by the sales mix

\(24\times X - 1656 = 0 \) (break-even)

\(24\times X = 1656\)

\(X = 1656 ÷ 24\)

\(X = 69\) total units including Products A, B, and C by the sales mix
Step 4: *(Because partial unit cannot be sold, it is rounded up to the next whole number.)*

- **Product A:** $69 \times \frac{1}{(4+1+1)} = 34.50 \approx 35 \text{ units}
- **Product B:** $69 \times \frac{1}{(4+1+1)} = 25.875 \approx 26 \text{ units}
- **Product C:** $69 \times \frac{1}{(4+1+1)} = 8.625 \approx 9 \text{ units}

Step 5: *(Proof)*

\[
(20 \times 35) + (25 \times 26) + (37 \times 9) = 700 + 650 + 333 = 1683
\]

$1683$ being the total contribution margin by the sales mix

$1656$ being the total fixed costs for the period

$1683 - 1656 = 27 \neq 0$ due to rounding, the break-even point by the current approach

**Suggested Approach**

Under the suggested approach of the study, Step 1 through Step 3 are the same as the current approach. Step 4 is the only step that is different from the current rounding. By the suggested approach of the study, in Step 4, rounding is done as follows;

Steps 1 through 3: *(same as the current approach)*

Step 4: *(Because partial unit cannot be sold, it is rounded up or down to a whole number.)*

- **Product A:** $69 \times \frac{1}{(4+1+1)} = 34.50 \approx 34 \text{ units}$
- **Product B:** $69 \times \frac{1}{(4+1+1)} = 25.875 \approx 26 \text{ units}
- **Product C:** $69 \times \frac{1}{(4+1+1)} = 8.625 \approx 9 \text{ units}$

Step 5: *(Proof)*

\[
(20 \times 34) + (25 \times 26) + (37 \times 9) = 680 + 650 + 333 = 1663
\]
$1663$ being the total contribution margin by the sales mix

$1656$ being the total fixed costs for the period

$1663 - 1656 = 7 \neq 0$ due to rounding, the break-even point, closer to zero than the current approach

**Logic of Rounding of Suggested Approach**

In Step 4, Products A, B, and C turn out to be partial numbers: $34.50$, $25.875$, and $8.625$, respectively. Because partial unit cannot be sold, it requires rounding to a whole number some way.

For the sake of simplicity of an example, one assumption is made for the logic of suggested rounding to work that the total number of units to be sold to break-even comes out a whole number, which is $X = 69$ in Step 3 in the example. Then, because sum of all the three numbers should amount back to the whole number $(34.50 + 25.875 + 8.625 = 69)$, sum of only decimals of the three numbers $(.50, .875, \text{ and } .625)$ should amount to a whole number $(.50 + .875 + .625 = 2)$, too.

Using this logic, instead of just rounding up the partial numbers to the next whole number $(34.50 \approx 35, 25.875 \approx 26, \text{ and } 8.625 \approx 9)$, decimal portions of the partial numbers can be re-distributed in such a way that makes all the partial numbers whole numbers and gets closer to zero profit, a break-even, from the positive side of profit.

**Procedure of Rounding of Suggested Approach**

In the suggested approach of rounding, the way to make all the partial numbers whole numbers by re-distributing decimal portions is as follows;

Process 1: The products are placed in the order of unit contribution margin, placing the lowest unit-contribution-margin product first and the highest unit-contribution-margin product last. From the created example, the order is Product A ($20$), Product B ($25$), and Product C ($37$).

Process 2: Find a product that has the lowest unit-contribution-margin and has a partial number in Step 4, and take off decimal portion of the partial number. It is Product A in the example and $.50$ is taken off of $34.50$, leaving the whole number $34$ ($34.50 - .50$) for Product A.

Process 3: Any portion of the $.50$ taken off in Process 2 is added to a product that has the highest unit-contribution-margin and a partial number, making this partial number a whole number. It is Product C to which $.375$ off of $.50$ is added, making $8.625$ the whole number $9$ ($=8.625+.375$) for Product C.
Process 4: The remaining .125, after .375 is used out of .50, is added to a product that has the next highest unit-contribution-margin and a partial number, making its partial number a whole number. It is Product B to which the remaining .125 off of .50 is added, making 25.875 the whole number 26 (=25.875+.125) for Product B.

Process continues until all the partial numbers become whole numbers.

Elaboration of Logic of Decimal Re-Distribution of Suggested Approach

The reason that the decimals of low unit-contribution-margin products are taken off and added to the decimals of high unit-contribution-margin products is to prevent the profit from falling below zero, a break-even point. By this reasoning, decimals cannot be re-distributed in an opposite direction, from high unit-contribution-margin product to low unit-contribution-margin product, because this will cause the profit to fall negative, which is below a break-even point.

This upward decimal re-distribution ensures that the mathematical answers (partial numbers) are turned into practical answers (whole numbers) without falling below a break-even line.

After the suggested rounding procedure is finished, sum of the whole numbers equals the total number of units to be sold to break-even (34+9+26=69=X), which is 1 less than sum of the whole numbers under the current approach (35+9+26=70). This shows that the suggested rounding approach finds a closer break-even point than the current approach by one unit of Product A for the specific example created.

Fine Tuning of Suggested Approach

It is possible to find a break-even point that is even closer to zero profit for the specific example.

First, find the differences of unit contribution margin between every two of the products. For the example of the study, they are 5 between Product A (20) and Product B (25), 17 between Product A (20) and Product C (37), and 12 between Product B (25) and Product C (37).

Second, find the differences that are equal to or less than 7, and choose the biggest one. Seven is the number by which the answer for a break-even point under the suggested approach exceeds a conceptual break-even point as shown in Step 5. For the example, it is 5 between Product A (20) and Product B (25) that meets this condition.

Third, take one unit off of 26 units of Product B and add this one unit to 34 units of Product A, making 35 units for Product A and 25 units for Product B. This time it is moved from a higher unit-contribution-margin product to a lower unit-contribution-margin product in order to decrease the profit as much as the difference of unit contribution margin between the two
products. The excess profit, 7, from zero profit under the suggested approach is further reduced by 5; now, the excess profit is down from 7 to 2 as shown below. The number 5 is the very difference of unit contribution margin between Product A and Product B.

Step 5: (Proof) – Suggested Approach

\[
(20 \times 35) + (25 \times 25) + (37 \times 9) = 700 + 625 + 333 = 1658
\]

$1658$ being the total contribution margin by the sales mix

$1656$ being the total fixed costs for the period

$1658 - 1656 = 2 \neq 0$ due to rounding, the break-even point, the closest to zero profit

From the partial numbers (34.50, 25.875, and 8.625 for Products A, B, and C, respectively), at the end, Product A is rounded up to 35 units, Product B rounded down to 25 units, and Product C rounded up to 9 units. These are the closest possible answers in units to a break-even point. The purpose of the suggested approach is to develop the current approach further to produce the closest possible answers in a CVP analysis.

Target Profit

Current Approach

Under the current approach, the Question, ‘What is the least number of units that should be sold to meet the target profit of $7,440 for the period?’ is answered as follows;

Step 1: \( Y \) being the total number of units to be sold to meet the target profit

\[
24 \text{ being the average contribution margin by the sales mix}
\]

\[
24 \cdot Y - 1656 = 7440
\]

\[
24 \cdot Y = 7440 + 1656
\]

\[
Y = \frac{7440 - 1656}{24} = 379
\]

Step 2: (Because partial unit cannot be sold, it is rounded up to the next whole number.)

Product A: \( 379 \times \frac{4}{4+1+1} = 189.50 \approx 190 \) units
Product B: $ \frac{3}{(4+1+1)} = 142.125 \approx 143 $ units

Product C: $ \frac{1}{(4+1+1)} = 47.375 \approx 48 $ units

$ 190 + 143 + 48 = 381 \neq 379 $ due to rounding, total units for target profit under the current approach

Step 3: (Proof)

+ Sales Revenue: $ (30 \times 190) + (42 \times 143) + (62 \times 48) = 5700 + 6006 + 2976 = 14682 $

− Variable Costs: $ (10 \times 190) + (17 \times 143) + (25 \times 48) = 1900 + 2431 + 1200 = 5531 $

$ = $ Contribution Margin: $ 14682 - 5531 = 9151 $

− Total Fixed Costs: $ 1656 $

$ = $ Profit: $ 9151 - 1656 = 7495 > 7440 $ due to rounding, target profit met under the current approach

**Suggested Approach**

The mechanism of suggested approach for a target profit is fundamentally the same as that for a break-even point. This is because a break-even point is one of target profit points, which is set at zero profit. Under the suggested approach, the answer for $ 7,440 target profit is found as follows;

Step 1: (Same as the current approach)

Step 2: (Because partial unit cannot be sold, it is rounded up or down to a whole number.)

Product A: $ \frac{4}{(4+1+1)} = 189.50 \approx 189 $ units

Product B: $ \frac{3}{(4+1+1)} = 142.125 \approx 142 $ units

Product C: $ \frac{1}{(4+1+1)} = 47.375 \approx 48 $ units

$ 189 + 142 + 48 = 379 $, total units for target profit under the suggested approach
Step 3: (Proof)

+ Sales Revenue: ($30\times189) + ($42\times142) + ($62\times48) = $5670 + $5964 + $2976 = $14610

− Variable Costs: ($10\times189) + ($17\times142) + ($25\times48) = $1890 + $2414 + $1200 = $5504

= Contribution Margin: $14610 − $5504 = $9106

− Total Fixed Costs: $1656

= Profit: $9106 − $1656 = $7450 > $7440 due to rounding, closer to the target profit than the current approach

Logic of Rounding of Suggested Approach

In Step 2, Products A, B, and C turn out to be partial numbers: 189.50, 142.125, and 47.375, respectively. Because partial unit cannot be sold, it requires rounding to a whole number some way.

For the sake of simplicity of an example, one assumption is made for the logic of suggested rounding to work that the total number of units to be sold to make the target profit comes out a whole number, which is $Y = 379$ in Step 1 in the example. Then, because sum of all the three numbers should amount back to the whole number (189.50+142.125+47.375=379), sum of only decimals of the three numbers (.50, .125, and .375) should amount to a whole number (.50+.125+.375=1), too.

Using this logic, instead of just rounding up the partial numbers to the next whole number (189.50≈190, 142.125≈143, and 47.375≈48), decimal portions of the partial numbers can be re-distributed in such a way that makes all the partial numbers whole numbers and meet the target profit as close as possible.

Procedure of Rounding of Suggested Approach

In the suggested approach of rounding, the way to make all the partial numbers whole numbers by re-distributing decimal portions is as follows;

Process 1: The products are placed in the order of unit contribution margin, placing the lowest unit-contribution-margin product first and the highest unit-contribution-margin product last. From the created example, the order is Product A ($20), Product B ($25), and Product C ($37).

Process 2: Find a product that has the lowest unit-contribution-margin and has a partial number in Step 2, and take off decimal portion of the partial number. It is Product A in the example and .50 is taken off of 189.50, leaving the whole number 189 (189.50−.50) for Product A.
Process 3: Any portion of the .50 taken off in Process 2 is added to a product that has the highest unit-contribution-margin and a partial number, making this partial number a whole number. It is Product C to which all .50 is added, making 47.375 still the partial number 47.875 (=47.375+.50) for Product C.

Process 4: Find a product that has the next lowest unit-contribution-margin and has a partial number in Step 2, and take off decimal portion of the partial number. It is Product B in the example and .125 is taken off of 142.125, leaving the whole number 142 (142.125−.125) for Product B.

Process 5: Any portion of the .125 taken off in Process 4 is added to a product that has the highest unit-contribution-margin and a partial number, making its partial number a whole number. It is still Product C to which all .125 is added, making 47.875 the whole number 48 (=47.875+.125) for Product C.

*Process continues until all the partial numbers become whole numbers.*

**Elaboration of Logic of Decimal Re-Distribution of Suggested Approach**

The reasoning for upward decimal re-distribution for a target profit is the same as that for a break-even point; that is to ensure that the mathematical answers (partial numbers) are turned into practical answers (whole numbers) without failing the target profit.

After the suggested rounding procedure is finished, sum of the whole numbers equals the total number of units to be sold to meet the target profit (189+142+48=379=Y), which is 2 less than the sum of the whole numbers under the current approach (190+143+48=381). This shows, again, that the suggested rounding approach meets the target profit closer than the current approach by two units: one less unit of Product A and one less unit of Product B. These are the closest answers that can be found for the target profit, and the suggested approach serves the purpose of finding the answers.

For the target profit, it does not take fine tuning for this specific example, but it could, in general, as in finding a break-even point under the suggest approach.

**Application of Suggested Approach to a Two-Product Company**

Finding a break-even point or target profit point for a two-product company under the suggested approach is as simple as the current approach with a little difference.

Using the following example of a two-product company, answers are found for a break-even point under both the current and suggested approaches. Then, the answers under each of the approaches are compared against each other.
Table 2. A Two-Product Company

<table>
<thead>
<tr>
<th>Product</th>
<th>Product A</th>
<th>Product B</th>
</tr>
</thead>
<tbody>
<tr>
<td>Unit Selling Price</td>
<td>$100</td>
<td>$130</td>
</tr>
<tr>
<td>Unit Variable Costs</td>
<td>60</td>
<td>100</td>
</tr>
<tr>
<td>Unit Contribution Margin</td>
<td>$40</td>
<td>$30</td>
</tr>
<tr>
<td>Sales Mix Ratio</td>
<td>3</td>
<td>2</td>
</tr>
</tbody>
</table>

Fixed Costs for the Period = $864

**Break-Even Point**

*Current Approach*

Under the current approach, the break-even point of the two-product company is calculated as follows;

Step 1: $(40\times3) + (30\times2) = 120 + 60 = 180$
Step 2: $180 \div (3+2) = 180 \div 5 = 36$

$36$ being the average contribution margin by the sales mix

Step 3: $X$ being the total number of units to be sold to break-even by the sales mix

$36\times X - 864 = 0$ (break-even)

$36\times X = 864$

$X = 864 \div 36$

$X = 24$ total units including Products A and B by the sales mix

Step 4: *(Because partial unit cannot be sold, it is rounded up to the next whole number.)*

Product A: $24 \times \frac{3}{(3+2)} = 14.40 \approx 15$ units

Product B: $24 \times \frac{2}{(3+2)} = 9.60 \approx 10$ units

Step 5: *(Proof)*

$(40\times15) + (30\times10) = 600 + 300 = 900$

$900$ being the total contribution margin by the sales mix

$864$ being the total fixed costs for the period

$900 - 864 = 36 \neq 0$ due to rounding, the break-even point by the current approach
Suggested Approach

Steps 1 through 3: *(same as the current approach)*

Step 4: *(Because partial unit cannot be sold, it is rounded up or down to a whole number.)*

\[
\text{Product A: } 24 \times \frac{9}{(9+2)} = 14.40 \approx 15 \text{units}
\]

\[
\text{Product B: } 24 \times \frac{2}{(9+2)} = 9.60 \approx 9 \text{units}
\]

Step 5: *(Proof)*

\[
($40 \times 15) + ($30 \times 9) = $600 + $270 = $870
\]

$870 being the total contribution margin by the sales mix

$864 being the total fixed costs for the period

\[
\$870 - \$864 = $6 \neq 0 \text{ due to rounding, the break-even point, closer to zero than the current approach}
\]

Once the total number of units sold to break-even, or to meet another target profit, comes out as a whole number, it only takes removing the decimal portion of partial number of low unit-contribution-margin product and adding it to the partial number of high unit-contribution-margin product. This way, the partial number for low unit-contribution-margin product is rounded down, and the partial number for high unit-contribution-margin product rounded up, making both partial numbers whole numbers. Sum of these two whole numbers for the two products should equal the total number of units sold to break-even (15+9=24), or to meet another target profit, for this company.

Discussion and Conclusion

Among the managerial accounting texts available in the market, in the chapter for a CVP analysis, many of the texts use the method of up-rounding partial numbers to whole numbers in finding the number of units for a break-even point or for a target profit, and some others do up-rounding or down-rounding partial numbers to whole numbers using the traditional .50 threshold.

As of the time when this study is carried out, no other methods are found yet that get to closer answers to a break-even point and a target profit than the current approach that is being taught. Thus, it is attempted in the study to develop a systematic approach to refining answers for the two basic questions in a CVP analysis: a break-even point and a target profit. It is
hoped, then, that the micro approach developed and introduced in this study help find closer answers to a break-even point and a target profit in a cost-volume-profit analysis.

Author’ Biography

Dr. Seung Hwan Kim got master’s and doctorate in accounting in 1999 and 2008, respectively, both from Southern Illinois University Carbondale. He currently teaches accounting at Indiana University of Pennsylvania. His main research interests lie in e-commerce, accounting information systems, and accounting education.

References

